This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

ANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STAN
IVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD UNIV
RARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRA
ITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY
IES · STANFORD RD UNIVERSITY LIBRARIES
RD UNIVERSITY ITY LIBRARIES · STANFORD
ANFORD UN UNIVERSITY LIBRARIES · STAN
IVERSITY LI LIBRARIES · STANFORD UNIV
RARIES · S STANFORD UNIVERSITY LIBRA
ITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY
IES · STANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES
RD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD
ANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STAN
IVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD UNIV
RARIES · STANF LIBRARIES · STANFORD UNIVERSITY LIBRA

Ellen Eyre Morgan Marshall
and
Stewart McColloch Marshall

A gift to the
Stanford University Libraries

IVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD UNIVE
IBRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBR
TANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STAN
· STANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES · S
D UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD U
Y LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY L
IVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD UNIVE
IBRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBR
TANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STAN
· STANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES · S
D UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD U
Y LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY L
IVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD UNIVE
IBRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBR
TANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STA

ANFORD UNIVERSITY LIBR
NIVERSITY LIBRARIES · ST
BRARIES · STANFORD UNIV
ITY LIBRARIES · STANFORD
IES · STANFORD
ORD UNIVERSITY
ANFORD UN
NIVERSITY LI
BRARIES · ST
ITY LIBRARIES · STANFORD
IES · STANFORD UNIVERSITY
ORD UNIVERSITY LIBRARIES
ANFORD UNIVERSITY LIBRARIES
NIVERSITY LIBRARIES · STANFORD
RARI UNIVERSITY

ANALYTICAL PRINCIPLES

OF THE

XPANSIVE STEAM ENGINE.

ANALYTICAL PRINCIPLES

OF THE

EXPANSIVE STEAM ENGINE.

ANALYTICAL PRINCIPLES

AND

PRACTICAL APPLICATION

OF THE

EXPANSIVE STEAM ENGINE,

AS EMPLOYED

IN PUMPING, MANUFACTURING, STEAM NAVIGATION, RAILWAY LOCOMOTION, &c.

AND IN WHICH THE PROPERTIES OF EXPANSION ARE DEVELOPED, AND A COMPARATIVE VIEW GIVEN OF THE ADVANTAGES TO BE OBTAINED BY THE ADOPTION OF ENGINES ON THIS PRINCIPLE.

WITH WOODCUTS ILLUSTRATIVE OF THE THEORY, AND ENGRAVINGS OF ENGINES OF THE MOST APPROVED ARRANGEMENTS OF PARTS.

By PAUL R. HODGE, C.E.

LONDON:
JOHN WILLIAMS & CO.,
ARCHITECTURAL AND ENGINEERING PUBLISHERS,
141, STRAND.

MDCCCXLIX.

RP

TJ 464
H 688

H 789

A

MEMORIAL OF RESPECT

TO

MY FELLOW LABOURERS IN PRACTICAL SCIENCE AND ART,

BY

THEIR OBEDIENT SERVANT,

THE AUTHOR.

PREFACE.

So much having been written on the principles of the Steam Engine, any further attempts to elucidate the laws which govern that mighty machine might be considered useless, particularly as there have been published up to the present date, nearly one hundred volumes on the subject. I, however, have, for the second time in my life, again put my ideas together, and, with all humbleness, give them to my fellow Engineers, hoping they may *cull* from these pages much that may be valuable and instructive.

This Treatise contains the results of many years of research and practical experience; and, I may add, that I had given up all idea of ever again writing on the Steam Engine, had it not been for the fact that, after an absence of nearly sixteen years, I returned again to my native country, and found that errors had become sanctified by long prescription, and had actually put on the appearance, and usurped the place of truth so strongly, that many Engineers in this country would not even believe what was actually being done by their neighbours, although they read the reports monthly for years. They had, therefore, put down the Cornish Engineers and Mine Agents as men publishing untruths, and not to be believed. I am, however, proud to say, that these truths are being recognised by Engineers generally, but not until they underwent an unrelenting ordeal of repudiation. To use the words of Scott Russell, "*The progress of the Steam Engine has been co-ordinate with the progress of the human mind in physical science.*" Although many of these valuable and progressive steps have had their birth in Cornwall, yet, if that county had coal-mines in close proximity to their copper and tin, I doubt not we should have remained ignorant of those beautiful principles which have been so fully and effectually carried out. My readers may not be aware that valuable mines were, twenty years ago, stopped in consequence of the great cost of drainage, and yet the Pumping Engines were then doing greater duty than many of the Factory Engines of Manchester at the present date: and the same mines have been again put to work, and have again assumed their former greatness; and that which seemed hopeless has been looked upon, in the present day, as a mere nothing: and all this has been carried out because the Engineers are pumping the same amount of water for less than one-sixth of the former cost.

Without hyperbole, it might be said the torch of science has again illumined the gloomy chambers of the earth, and shed its lustre on many a happy homestead, and cheered many thousand hearts.

We have still further facts as to the difficulty of introducing the Expansive Principle into Her Majesty's Navy. The Secretary of the Navy, but a few years since, obtained with great difficulty, the consent of the Lords of that Board to have these improvements adapted to Her Majesty's war-ship *Echo*. The order was forthwith executed, and forthwith condemned, on the testimony of those who have cost our nation twenty millions sterling in foolish experiments. The Cheap Cornish System was pronounced unsuited to the wealth of the British Navy, and the Symondsian System again resumed its extravagant sway.

Truth must, therefore, treat events like these as the dark days and evil times of Steam obscuration. The author feels proud to say, that these valuable principles are now being appreciated and carried out by

INDEX OF CONTENTS.

SECTION I.

PRESSURE OF ELASTIC FLUIDS.

PAGE
Law governing their expansion in a close vessel 1
The pressure is inversely as the space occupied . . . . . „
This law first announced by Boyle and enforced by Marriotte 2
Law, inference and specific expression . . . . „
Temperature not yet considered: air adopted as the basis of the investigation . . „
Fig. 1. illustrating theory and its effects . . . . 3—4
Analogy between the pressures and spaces when the air is expanded . . . 5
Fig. 1^{a}. shows the effect of compression . . . . „
Computation of ditto . . . . „
Exhibited symbolically 6
Analogies: fig. 2. showing effect on a moveable air-tight piston . . . 7
Notation extended to suit diagram: analogies 8
Equations: curve of expansion assimilated to the common hyperbola . . . 9
Advantages of such a coincidence: how hyperbola may be constructed . . . „
Best method of finding the points in the curve 10
Analogy and equivalent equation identical with Marriotte's law . . . 10—11
No other method of construction need be resorted to: general law . . . 11
Fig. 4. showing how the hyperbola exhibits the pressure by ordinates . . . 12
Atmospheric air being compressed into a less space, the increase of pressure is more rapid than its diminution by expansion 13
Analogies . . . . „
Elasticity increased more and more rapidly as compression proceeds . . . 14
Fluxional analysis necessary to determine mean pressure . . . . . „
Rule for approximating thereto . . . . ibid.—15
Application of the rule by division of the piston's motion into intervals . . 15
These results are in excess when compared to fluxional analysis . . . 16

SECTION II.

PROPORTION OF STEAM WHEN IN CONTACT WITH LIQUID.

Biot's equation applicable to the scale of temperature, but almost useless for calculating the steam-engine 17
Other formulæ which do not extend individually through a sufficient range, yet, when taken in connexion, are sufficient to construct a table on Fahrenheit's scale . 18

PAGE
Southern's formula applicable to a temperature below the boiling point . . 18
Practical rule for reducing ditto . . . . ibid.—19
Tredgold's formula, as corrected by Mellett, carries us up to four atmospheres . . 19
Ditto, adapted to logarithmic operation . . . . „
Practical rule . . . . „
Example 20
Practical rule by common arithmetic . . . . „
Ditto, by Logarithm . . . . „
Comte de Pambour's formula expressed logarithmically . . . . . . „
Arago and Dulong's formula extend the table to fifty atmospheres . . . . „
Practical rule for ditto, expressed also logarithmically . . . . . . „
Practical rule and example 22
They are found to agree with the experiments of others . . . . . „
Table of pressures found by combining the four formulæ 23
Equation for Southern's formula . . . . „
Practical rule and example . . . . ibid.—24
Transformation of Tredgold and Mellett's formulæ . . . . . . „
Practical rule, according to ditto, for finding the temperature due to a given pressure . „
Transposition of Comte de Pambour's equation 25
Rule for reducing ditto . . . . „
Example . . . . „
Transformation of Arago and Dulong's formula 26
Rule for ditto . . . . „
Equations for the temperature by juxta-position of the four equations . . . „
The custom of expressing the elastic force of steam in inches of the Mercurial column must be considered in fully applying these formulæ 27
Equation for calculating ditto in Southern's formula . . . . . . „
Relative specific gravities of water and mercury . . . . . . „
Rule founded on such data 28
Rule and equations giving the elastic force in inches of mercury in substitution of Southern's equation . . . . „
Practical rule for ditto 29
Another form of equation . . . . „
Practical rule . . . . „
Examples . . . . ibid.—30
The rules verify each other . . . . „
Tredgold and Mellett's formulæ modified to inches of mercury . . . . . 30—31
Practical rule . . . . „
Example . . . . „
Arago and Dulong's equation modified to inches of mercury 32
Adapted to logarithms . . . . „
Practical rule . . . . „
Example . . . . „
Tredgold's formula sufficient up to four atmospheres 33

PAGE
Arago and Dulong's formulæ do. for higher points 33
Equation for Tredgold's formula . . . . „
Practical rule for do. . . . . „
Example 34
Dulong and Arago's formulæ are easily adapted to the higher pressures . . „
On the space which steam occupies under pressure compared to that of the water from which it is generated . . . . „
The law of Marriotte does not apply unless combined with the property discovered by Gay Lussac . . . . „
What that property is 35
Rule and equation deduced therefrom . . . . „
Result of the combination by equations 36
Practical rule . . . . „
Example under Dulong and Arago's rule for determining the pressure due to a given temperature 37
Rule for finding the volume due to the pressure . . . . . . „
Do. for finding the volume due to the temperature . . . . . . ibid.—38
Difference between the results of the Comte de Pambour's formulæ and the other four explained 38
In using steam expansively these formulæ do not give correct results, but are near enough for practice . . . . „
Equation for the volume in terms of the pressure . . . . . . „
Practical rule 39
Example . . . . „
Formula for the volume in terms of the temperature . . . . . . „
Equation when the pressure exceeds four atmospheres . . . . . . „
On the importance of the preceding methods 40
Pneumatic law, by which elastic fluids, under a given pressure, flow into a vacuum due to the weight of an uniform column of such fluid pressing on its base . „
Formula and equation founded therein . . . . „
Practical rule . . . . „
Do., expressed logarithmically 41
Practical rule for finding the height of the column in part . . . . . „
Example . . . . „
Result . . . . „
Example for finding the velocity with which steam will flow into a vacuum . . 42
Forms of equation when the force is expressed in inches of mercury . . . 42—43
Do., by reference to the laws of falling bodies 43
Practical rule 44
Example . . . . „
No allowance here made for loss by passing through contested apertures . . „
Temperature and elastic force must depend on each other 45
The relation which exists between pressure, temperature, and density, ceases when the steam is separated from the liquid . . . . . . „

SECTION III.

EXPANSION OF STEAM WHEN SEPARATED FROM THE LIQUID.

PAGE

Steam separated from the water producing it, obeys the same laws of elasticity and density as air . . . 46

General law of Boyle as confirmed by Marriotte . . . ,,

The subsequent theory here annunciated depends on this law . . . 47

Gay Lussare's general law of elastic fluid . . . ,,

Definition of terms peculiar to the disquisition . . . ,,

Fig. 5, for showing the principle of expansion in reference to steam alone . . . 48

Curve of expansion with respect to fluids generally applied to steam exclusively by equation . . . 49

Equation (A) . . . ,,

Fig. 6, illustration showing curve of expansion . . . 50

Fig. 7, showing how the curve of expansion gives the force at any point of dilatation . . . 51

Fig. 8, showing the rate of expansion . . . 52

Proportions of expansion to the spaces . . . 53

Analogies for the force in the several spaces . . . ,,

Example . . . 54

Numerical value of ordinates or pressures by hyperbolic equation . . . 55

Fig. 9 delineates the curve of expansion whereby the decrease of force may be traced . . . ,,

The methods divest the subject of the intricacy and mystery usually ascribed to it . . . 56

Example to obtain closer approximation . . . ,,

Table of analogies for finding the decrements of pressure . . . 57—58

Table of logarithmic difference for the foregoing example . . . 59

Method of using the differences . . . 60

Arrangement for computation of the entire series from 10 to 40 . . . 60—61

Equation for finding the mean pressure . . . ,,

Example of further application of the rule at page 12 . . . 62

Method of obtaining the index of expansion . . . 62—63

Mode of assigning ditto, and examples to determine the series of terms for the pressure . . . 63

The mean pressure alone being required, differential analysis, although complicated, is necessary . . . 64

Notation for such analysis . . . ibid to 66

Equations (B and C) . . . ,,

Practical rule . . . ,,

Example . . . ,,

Value of total mean force . . . 67

Equation (D) determining mean pressure . . . 68

Practical rule for ditto . . . ,,

Ditto for determining to what degree of expansion we should proceed to obtain any mean pressure . . . ,,

SECTION IV.

SINGLE-ACTING ENGINE WORKING BY EXPANSION AND CONDENSATION.

PAGE
The consideration heretofore had no reference to form of engine 78
Enumeration of the different kinds of expansive engines 79
The mean effective force demonstrated by friction, leakage, cooling, and other causes . „
Their total amount about four-tenths of the force in the boiler . . . 80
Rule for calculating the power of single-acting engine . . . . . „
Example . . . . „

SECTION V.

THE DOUBLE-ACTING ENGINE WORKING BY EXPANSION AND CONDENSATION.

The double-acting engine developes twice the power, and requires twice the steam . 81
Principle of operation described 82
Method of computing the power by equation (G) . . . . . . „
Practical rule 83
Equation (H) modifying equation G . . . . „
Practical rule for obtaining the effect in horse power . . . . . . „
Equation (J) adapting the formula to logarithms 84
Practical rule . . . . „
Example . . . . 84—85
Allowance must be made for imperfect condensation 85
Result of both rules . . . . „
To ascertain the proper point of interception 85
The effect of varying ditto . . . . 86—87
Equation K developes an important fact, simplifying the calculation of power . . „
Equation L, simplifying the preceding . . . . „
Results by common arithmetic 88
Calculation repeated under equation K 90
About two-sevenths of the stroke the best point of cut-off 91
Equation M, for indicating the horse power . . . . „
„ N, ditto . . . . „
Practical rule 92
Calculation of power, how simplified, by cutting off at two-sevenths of the stroke . „
On what three data the power depends . . . . „
Rules for finding the third, any two being given 93
Equation for analysing the general equation, in reference to the diameter of the cylinder 94
Practical rule 95
Example . . . . 94—95
Equation for determining the mean pressure to the entire effect . . . . „
Practical rule 97
Example . . . . „
Equation for determining the velocity of the piston 97
Practical rule 98

PAGE

Example 99

Possessing the power of regulating the generation of the steam, we may modify the terminal pressure by varying the point of cut-off . . . . . ,,

Why the terminal pressure should never be less than sufficient to work the engine . 99

Rules for determining the point of cut-off with a given initial pressure to obtain a given means . . . . 99—100

Fig. 10 illustrating diagram 101

How uniformity of motion is maintained under such variation of motive force . . 102—104

Cases illustrating mean pressure in cylinders of different lengths 104

With the same capacity, the shorter the stroke the greater the power 105

No settled rules as regards length compared to diameter . . . . . ,,

Cooling surface a material consideration . . . . ,,

Two lengths to one diameter the best proportions 106

Rule on which this proportion is founded . . . . ,,

Further comparison between long and short stroke engines 107

Cases showing where the long stroke has the advantage in power . . . . 107—108

But a greater proportion of steam is expended . . . . ibid.—109

Allowances must be made for imperfect vacuum 110

Equation for determining how much is lost thereby . . . . . . ,,

Practical rule 111

Example . . . . ,,

Other resistances must be determined by observation 112

SECTION VI.

THE DOUBLE-ACTING NON-CONDENSING ENGINE WORKING BY EXPANSION.

The terminal pressure must be sufficient to overcome the atmospheric resistance, together with friction and other opposing forces 113

Theory alone cannot decide the limit of terminal pressure 114

Rule for approximating thereto . . . . ,,

Equations for determining the relative expansion 115

Example . . . . ,,

Formulæ for determining the cut-off point . . . . 116—118

Practical rule . . . . ,,

Example . . . . ,,

SECTION VII.

THE COMBINED CYLINDER EXPANSIVE ENGINE.—DESCRIPTION OF WOOLF'S ARRANGEMENT.

Illustrated by diagram (Fig. 11) 120

Force of the steam never less than equal to the full pressure on the smaller piston . 121

Formulæ for computing the relative capacities of the cylinders 122

Effect on the larger cylinder is wholly additional to that produced on the smaller . . 123

Various methods of computing the power . . . . ,,

Examples in actual practice, selected to illustrate the rules . . . . ,,

PAGE
Consumption of fuel in the example . . . 124
Do. of steam in ditto . . . „
Comparative capacity of the cylinder in ditto . . . „
Capacity of passages to be calculated in order to determine the true relative expansion . 125
Fig. 12 illustrating diagram . . . 127
Terms of series for calculating the pressures . . . „
Analogies for determining the pressures at every tenth of the stroke . . . ibid.—128
To find the mean pressure arithmetically . . . „
By equation . . . 129
Practical rule . . . „
To find the mean pressure between the pistons . . . „
Equation for calculating the momentum, or mechanical effect . . . 130
Practical rule . . . „
Examples . . . „
To find the power of each cylinder . . . 131
By another method . . . „
Practical rule for finding the power in the lesser cylinder . . . 132
Ditto for the gross power . . . „
By a third method . . . „
Comparison of the power of the combined cylinder engine with that of a single cylinder engine acting expansively . . . 133
Formulæ for the mean pressure . . . 134
Logarithm to find the gross power . . . 135
Advantage of the single cylinder over the combined cylinder engine . . . „
Sims' combined cylinder engine, described and illustrated by fig. 13 . . . 136—138
Fig. 14, geometrical method of determining the relative areas of the cylinders . . 138—139
Equal motive power in the ascending and descending stroke is obtained by proportioning the areas . . . 140
Rules for calculating the power . . . 141
Formulæ for the mean pressure . . . 142
Example . . . „
Computation of proportional spaces during a series of movements . . . 143
Proportion of pressure for each space . . . „
Mean pressure . . . „
Method of finding do. arithmetically . . . 144
Ditto, ditto, by equations . . . 145
Formulæ for effective force . . . „
For the larger cylinder . . . 146
For the smaller . . . „
For the combined, or total momentum . . . „
Equation for producing equal momentum in both up and down strokes . . . „
The diameter of one cylinder being given, to find that of the larger one to produce equal momentum . . . 147
The larger being given, to find the smaller to produce the same equal momentum . „
Formulæ for the same . . . 148—149

Practical rules
Example . . . . „
Greater economy of steam than in Woolf's system . . . . 150—151
Double expansion practicable by expanding in the lesser cylinder 151
Formula for calculating the power under double expansion . . . . „
Example . . . . „
Mean purpose 152
Relative expansion 153
Comparison of the engines 154
Advantages very considerable in single cylinder engines . . . . „
Power not so easily calculated . . . . „
Three-fifths of the steam effective 155
General example . . . . „
General formula for determining the power in lbs. 156
Logarithm for do. . . . . „
Pressure required to be maintained on the boiler 157
Mode of computing mechanical effect . . . . „
Economy of earlier cut-off 158
Its limits . . . . „
Equation for assigning initial pressure . . . . „
By logarithm 159
Lower initial pressure requires longer cut-off . . . . „
Logarithm proving this, and the increased consumption of steam consequent on lower initial pressure 160
Equation proving do. . . . . „
Woolf's engine—misconception which exists regarding it 161
The idea it propounds can never be realised practically . . . . „
Nor is the force of the expanding steam wholly additional . . . . „
Advantages more imaginary than real 162
Sims' engine—the same objections do not exist . . . . 162—163
Half the steam only consumed compared to Woolf's . . . . „
Greater simplicity of construction 163
Macnaught's arrangement of Woolf's principle 164

SECTION VIII.

SIMS' WHEEL ENGINE.

Description 165
Fig. 15 diagram for finding the mean average 166
Equation for investigating the power 167
Mechanical law on which the power depends . . . . „
Symbols for determining the mean effective radius 168
Rules for finding the mechanical effect . . . . ibid.—169
Value of the mean radius—space traversed by the moving power 170
Further explanation of rules for finding the power . . . . 170—171
Calculations proved by casual experiment 171
This engine works with greater ease and economy than any other 172

REFERENCE TO THE PLATES.

We have, in the preceding sections, treated of the Steam Engine, purely with reference to theoretical principles; nor is it our intention to go minutely into detail in the following description.

Our work is intended for those who have derived from other quarters a competent knowledge of the general construction of those modifications of the Steam Engine which have obtained in practice, but who are not so well acquainted with the best methods of availing themselves of the value of the elastic properties of steam.

We therefore submit the following plates as specimens of the three great applications of the Steam Engine, arranged with a view to the development of the expansive properties; and we need hardly state, that to the class of readers to which our work is directed, minute descriptions would be wholly useless, and that the illustrations will furnish nearly all the information necessary to be given.

PLATE I.

Portable condensing expansive engine, by the author, many of which are in actual operation, and are working with a consumption of fuel of three lbs. of coal per horse power per hour. The construction of this engine is very simple; it is provided with the double beat valve, and a separate double beat expansion valve. Worked by a cam motion.

PLATES II. AND III.

SIMS' DOUBLE CYLINDER EXPANSIVE STEAM ENGINE.

At page 136, and consecutively, the reader will find a general description of this invention, and an analysis of its properties, and these plates represent an example, as constructed by Mr. William West, of St. Blazey, Cornwall; plate 2 being an elevation, and plate 3 a plan thereof. The engine, in this instance, is constructed for drawing minerals.

PLATE IV.

SIMS' DOUBLE CYLINDER COMBINED PUMPING ENGINE. *By the Author.*

This engine, erected by the author for the Guadal Canal Silver Mines, near Seville, presents a novel, and in this instance, more convenient, mode of arranging the beam. This is (as will at once be perceived) placed under the engine, instead of above, as in the ordinary method of construction. The piston-rod works through a stuffing-box, at the bottom of the larger cylinder, and the air-pump is worked by side rods, and a cross head.

The other arrangements will be sufficiently intelligible without further reference.

PLATE V.

ENGINES OF THE PEKIN AND SULTAN STEAM VESSELS, CONSTRUCTED FOR THE ORIENTAL STEAM NAVIGATION COMPANY. *By Messrs. Tod and M'Gregor, Glasgow.*

The Scottish engineers were among the first to perceive the value of expansive steam, as applied to marine engines; and the firm who constructed our example were early in the field in adopting it; and we may say, that owing to their judicious use of moderately higher pressure worked expansively, and to the compactness of the arrangement of their engines, they have obtained a deservedly high reputation as engineers.

The engines here given are of the direct kind, that is to say, the parts move upwards or downwards simultaneously, a contradistinction to the beam engine, where, at any given moment, certain of the parts are moving in opposite directions. It is unnecessary to trouble our readers with enumeration of details where the form and action are so self-evident. We shall merely, therefore, give a brief description of the way in which the cut-off is effected.

The steam is admitted, and presses on the upper surface of one part, and the under surface of the other part, of the double beat valve. This valve is worked by a cam motion, which consists of a series of cams, four in number. The rotation of the axis causes the cams to act upon the roller, and so lift the valve and allow it to close again during any portion of the revolution. When the vessel is running light, or with favourable winds and weather, the roller is shifted so as to be acted upon by the cam, which cuts off the steam earliest, and therefore carries the expansion to the greatest extent; while with adverse weather, or deeply laden, the expansion is reduced to a minimum by the same adjustment, and the direct effect and full force of the steam from the boiler is prolonged proportionably.

PLATES VI. VII. AND VIII.

PASSENGER ENGINE AS MADE FOR THE SHEFFIELD AND MANCHESTER RAILWAY.

By Messrs. Sharp, Brothers, and Co., Manchester.

These plates are introduced as an example of the locomotive engines working expansively. The expansion in these engines is effected by the slide valve alone, and not, as in our other instances, by a separate expansion valve. The valves have for this purpose what is termed an increased lap, that is to say, the faces are wider than for non-expansive engines. The effect of this extension of the faces is to close the passage, and therefore cut off the steam from the boiler before the piston has completed its stroke, and an increase of lap would obviously give the power of cutting off as much earlier as may be desired.

SECTION I.

PRESSURE OF ELASTIC FLUIDS.

IF a given quantity, or a given weight of an elastic fluid, be confined in a close vessel of any form whatever, every particle throughout the mass of fluid so contained will sustain an equal pressure, upwards, downwards, and laterally, and the fluid will press equally on every point of the containing surface. If the containing vessel be enlarged in its dimensions, while the quantity of fluid remains the same, the fluid will not continue to occupy the original space, but will instantly diffuse itself throughout the enlarged vessel, until it is again retained in a state of equilibrium, by coming into contact with, and pressing with the same force on every point of the interior surface of the vessel. In this case also, every particle throughout the mass of fluid sustains an equal pressure in every direction, and the whole is in a state of perfect quiescence; but the intensity of pressure is not the same in both cases, being greater as the capacity of the vessel is less, and less as the capacity of the vessel is greater; that is,—

> *The pressure of a given quantity, or a given weight of an elastic fluid, on the interior surface of the vessel that contains it, is inversely proportional to the capacity of the vessel, or the space which it is made to occupy.*

This is a general law of elastic fluids, found to obtain in all cases where the temperature remains unaltered. It was first announced by Boyle, and afterwards abundantly verified by the experiments of Mariotte, and it is that which is made the basis of our investigations in reference to the properties and effects of all elastic fluids. From this law we infer,—

> *That the pressure of an elastic fluid, in every state of its density, upon the surface of the vessel that contains it, when multiplied by, or when drawn into the space which it occupies in any state, is always equal to the same constant quantity; that is, the space which the fluid occupies under any specified degree of pressure, when drawn into or multiplied by the pressure, is in every instance of the same value.*

This is a law of very great importance in the general doctrine of elastic fluids, and other branches of the physical sciences, and since its symbolical representation is remarkably simple, we cannot do better than to exhibit it in this place, and for this purpose,—

Put $C =$ the capacity of the vessel containing fluid of the pressure P,

and $C' =$ the capacity of another vessel containing fluid of the pressure P'.

Then by an inference deduced from the above law of elastic fluids, we obtain the following specific expression, viz.—

capacity of the vessel C *multiplied by the pressure* P, *is equal to the capacity of the vessel* C′ *multiplied by the pressure* P′; *that is,*

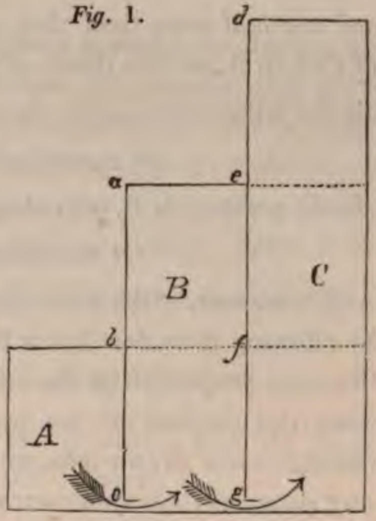
$$C \times P = C' \times P'.$$

And this principle holds good, whatever may be the space which the same quantity of the same fluid is made to occupy.

We here suppose that the temperature of the fluid remains the same for all possible changes in the volume and density, a supposition however, which is not strictly in accordance with the actual state of things, but such as may nevertheless be admitted in practice, without leading to any appreciable deviation from the truth.

The principle here stated is common to all elastic fluids, and extends even to the gases, but as atmospheric air is the best known to us, we think proper to adopt it as the basis of our investigations; and for this purpose, let A, B and C *fig.* 1, be three cylindrical vessels placed in close contact with each other, and all of the same diameter, but having their lengths in any specified proportion at pleasure, provided they are commensurable with each other, a condition which is necessary for the sake of simplicity; and moreover, let the three vessels mutually communicate with each other by means of the small openings or apertures at *c* and *g*; that is, the first opening to the second at *c*, and the second opening to the third at *g*.

Fig. 1.



Let the lengths of the vessels in the present instance, be to each other as the numbers 1, 2, and 3; then will the spaces or capacities of A, B, and C be exactly in the same proportion, for since the diameters are assumed as equal, the capacities by the rules of geometry, must vary directly as the altitudes or heights.

In the first place, conceive the vessel A, which is the smallest of the three, to be filled with air of the same density as the atmosphere in its natural state, or that surrounding the earth in immediate contiguity with its surface, or but little elevated above it; then will the pressure on every square inch of the containing surface be equal to fifteen avoirdupoise pounds; that is, the air within the vessel A exerts a pressure of fifteen pounds on every square inch of the interior surface, although it is entirely cut off from all communication with the external air; consequently, if the pressure of the outer air were wholly removed from the exterior surface of the vessel, that within it would exert a force of fifteen pounds on each square inch of surface, by endeavouring to expand itself into a larger space; from this it is obvious, that the vessel must resist with an equal force, in order to confine the air to its original volume.

In the next place, let the whole mass of air which is originally contained in the vessel A, be made to flow into the vessel B by opening the small aperture at *c*; then, since the capacity of the vessel B is double the capacity of the vessel A, it follows from the general law above enunciated, that the pressure of the air in B will be only half the pressure that it exerted in A, namely, seven and a half pounds upon the square inch, for according to the general law,—

The pressures are inversely as the space which the fluid occupies.

Hence it is as follows:—

As the pressure in A, *is to the pressure in* B, *so is the space* B, *to the space* A.

Or numerically thus; 15 : 7.5 : : 2 : 1, as stated above.

Again, let the whole mass of air which is now contained in the vessel B, be made to flow into the vessel C, by opening the small aperture at *g*; then, since the capacity of the vessel C is three times the capacity of the vessel A, and one half more than that of B, the pressure in C will be only one third of that in A, or two thirds of that in B; hence we have,

As the pressure in A, *is to the pressure in* C, *so is the space* C, *to the space* A.

Or numerically thus: 15 : 5 : : 3 : 1; or,

As the pressure in B, *is to the pressure in* C, *so is the space* C, *to the space* B.

Or numerically thus; 7.5 : 2.5 : : 3 : 2.

And moreover, if the same quantity or weight of air be still farther expanded, by allowing it to flow into a larger vessel, the pressure will be diminished in the same proportion as the capacity of the vessel is increased; but in every case, the product of the pressure of the air, whatever may be its state of density, when drawn into, or multiplied by the space which it occupies in any state, will always be expressed by the same number; or which amounts to the same thing, it will always indicate the same momentum.

But taking another view of the case, if the air of atmospheric density, which originally fills the vessel A, were permitted to expand itself so as to fill the two vessels A and B at one and the same time, every particle of the expanded mass would still be equally pressed in all directions, and it would press equally on every point of the containing surface, which would now be the interior of both the vessels A and B.

Here it is manifest from our general law of expansion, that in this case, the pressure would only be one third of what it was, when the same quantity of air was contained in the vessel A alone; for since the vessel B is double of A, it is obvious that the two vessels A and B together, must be equal to three times A, and because the pressures exerted by the same quantity of air when occupying different spaces are inversely as the spaces; we have as follows,—

As the pressure in A, *is to the pressure in* A + B, *so is the space* A + B, *to the space* A.

Or numerically thus; 15 : 5 : : 1 + 2 : 1.

Again, if the air which is now contained in the vessels A and B, be permitted still farther to expand itself, so as also to fill the vessel C, then every particle of the expanded air which is now diffused throughout the three vessels, A, B, and C, is equally pressed in all directions, and the air thus diffused, presses equally on every point of the containing surface, which is now that of all the three vessels, A, B, and C; but in this case, the pressure will be only one sixth part of what it was when the same quantity of air was contained in the vessel A alone, being now diffused into six times the original volume; for it is obvious from the assumed relative capacities of the vessels, that all the three taken together, are equal to six times the capacity of the vessel A, or that which the air occupied when it was of atmospheric density; and according to the foregoing general law, the pressures of the same quantity of air, are inversely as the spaces which it is made to occupy; hence it is,—

As the pressure in A, *is to the pressure in* A + B + C, *so is the space* A + B + C, *to the space* A.

Or numerically thus; 15 : 2.5 : : 1+2+3 : 1.

This is the analogy which obtains between the pressures and the spaces, when the air is so expanded as to fill all the three vessels.

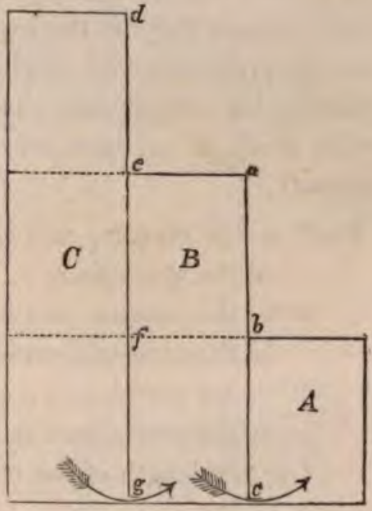


Fig. 1^a.

But on the other hand, if we suppose C *fig.* 1^a the largest of the three vessels, to be filled in the first place with air of atmospheric density, and the whole mass to be first compressed into the vessel B, and afterwards into the vessel A, then the relation between the pressures and the volumes will still obtain, according as we have expressed it by the foregoing general law, but in a contrary order from that which took place, when the air expanded from a smaller vessel into a larger one, the pressure in this case increasing as the space diminishes. Thus, when the whole of the air in the vessel C, which in its first state is of atmospheric density, is compressed into the vessel B, which is equal in capacity to two thirds of C, the pressure on a square inch of the interior surface of B, must be one half more than it was when the same quantity of air was contained in the vessel C; consequently, the pressure on a square inch in B, is 22.5 pounds; for, by the general law, it is—

As the space B, *is to the space* C, *so is the pressure in* C, *to the pressure in* B.

Or numerically thus; 2 : 3 : : 15 : 22.5.

Again, let the whole mass of air be compressed into the vessel A, which by the construction, is equal in capacity to one half of B, and one third of C; then will the pressure on a square inch of surface be equal to three atmospheres, or forty-five pounds; for according to the fundamental law, the pressures are inversely as the spaces occupied by the fluid; consequently, the pressure in A is double of the pressure in B, and triple of that in C; for we have,

As the space A, *is to the space* B, *so is the pressure in* B, *to the pressure in* A.

Or numerically thus; $1 : 2 :: 22.5 : 45$; and again,

As the space A, *is to the space* C, *so is the pressure in* C, *to the pressure in* A.

Or numerically thus; $1 : 3 :: 15 : 45$.

Since the three vessels in question are all of the same diameter, it is obvious from the principles of solid geometry, that their respective capacities are proportional to their lengths; it will therefore be sufficient in applying the general law, to consider the lengths of the vessels only, as by so doing, it will greatly simplify the expressions, and conduct us to a more interesting and comprehensive view of the subject. In order therefore, to exhibit symbolically the relation that obtains between the pressure of the fluid and the space which it occupies, we must adopt some such notation as that which follows:

Put $P =$ the pressure on a square inch of surface in the vessel A, viz., that of the atmosphere;

$P' =$ the pressure on a square inch of surface in the vessel B, viz., that of the atmosphere reduced in proportion to the increase of space;

$P'' =$ the pressure on a square inch of surface in the vessel C, viz., that of the atmosphere reduced as in the case of the vessel B;

$l =$ the length of the vessel A, in which the pressure on a square inch is P;

$l' =$ the length of the vessel B, in which the pressure on a square inch is P';

$l'' =$ the length of the vessel C, in which the pressure on a square inch is P''.

Then according to the general law of elastic fluids, we have the following analogies for the respective pressures in the vessels B and C, that in the vessel A being equal to the atmosphere in its usual state of density.

Thus we have, $l' : l :: P : P'$; that is, as the length of B, is to the length of A; so is the pressure in A, to the pressure in B; and, again, it is $l'' : l :: P : P''$; that is, as the length of C, is to the length of A; so is the pressure in A, to the pressure in C. Therefore, by working out these analogies in reference to P' and P'', we obtain as follows, viz:—

$$P' = \frac{l \times P,}{l'}$$

which in a specific form becomes,

$$\textit{pressure in } B = \frac{\textit{length of } A \times \textit{pressure in } A}{\textit{length of } B}$$

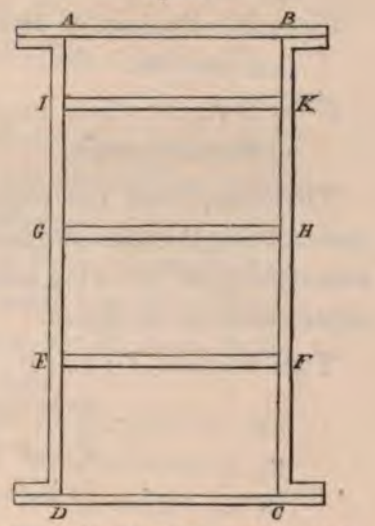
$$P'' = \frac{l \times P}{l''}$$

which in a specific form becomes,

$$\textit{pressure in } C = \frac{\textit{length of } A \times \textit{pressure in } A}{\textit{length of } C}$$

Now, if we carefully examine the composition of these results in the general form, viz., $P' = \frac{l \times P}{l'}$, and $P'' = \frac{l \times P}{l''}$; it will readily appear that each of them becomes assimilated to the equation of the common rectangular hyperbola when referred to the asymptotes, the quantities P, P' and P'' representing the ordinates, and l, l' and l'' the corresponding abscissas; but in order to render this still more evident, it will be convenient, instead of three separate vessels of different capacities, to consider the dilatations of the fluid as taking place in the same vessel, fitted with a moveable air-tight piston.

Fig. 2.



Let A B C D *fig.* 2, be such a vessel as is here proposed, the form being cylindrical, and the diameter uniform throughout the length, and this vessel is fitted with a moveable air-tight piston, as represented by the different positions in the figure, as at E F, G H and I K, thereby separating the vessel into the several spaces represented by C D E F, C D G H, C D I K and C D A B which spaces are successively filled by the fluid under different degrees of dilatation.

Conceive the lower space C D E F cut off by the first position of the moveable piston to be filled with air of atmospheric density, while the air is entirely excluded from the upper or complemental space A E F B, and at the same time, let the vessel be supposed to be wholly unaffected by the exterior atmosphere. Then it is obvious, that since the piston E F is air-tight but moveable, being unresisted in its motion upwards, the air below it by endeavouring to expand itself with a force or pressure of fifteen pounds upon the square inch, will produce motion in E F, and enlarge the space below it, and this enlargement will continue as long as the expansive force of the air is sufficient to move the piston, or until the motion is totally restrained, by the piston coming in contact with the top of the vessel, in which case, the vessel will be full of air under a high degree of dilatation. Therefore by repeating our notation, and extending it so as to suit the present diagram, we have

P = the pressure of the air in the space C D E F, being that of the atmosphere in its usual state;

P' = the pressure of the air in the space C D G H, being that of the air in the first stage of dilatation;

P'' = the pressure of the air in the space C D I K, being that of the air in the second stage of dilatation;

P''' = the pressure of the air in the space C D A B, being that which the air exerts when diffused throughout the vessel;

l = D E, the part of the vessel occupied with air of atmospheric density;

l' = D G, the part of the vessel occupied with air in the first stage of dilatation;

l'' = D I, the part of the vessel occupied with air in the second stage of dilatation;

l''' = D A, the whole length of the vessel, or the part of it through which the air expands.

Therefore, since the capacities are proportional to the lengths, the diameters being the same, we have by the general law of elastic fluids previously enunciated, the following analogies for the pressures in the several spaces as represented in the figure.

Thus we have, $l' : l :: P : P'$; that is, as the length l', is to the length l; so is the pressure P, to the pressure P';

" " $l'' : l :: P : P''$; that is, as the length l'', is to the length l; so is the pressure P, to the pressure P'';

" " $l''' : l :: P : P'''$; that is, as the length l''', is to the length l; so is the pressure P, to the pressure P'''.

And when these analogies are reduced to their equivalent equations, they severally assume the following forms, viz:—

$$P' = \frac{l \times P;}{l'}$$

which in a specific form becomes,

$$\text{pressure in C D G H,} = \frac{\textit{length } l \times \textit{pressure in } \text{C D E F}}{\textit{length } l'}$$

$$P'' = \frac{l \times P;}{l''}$$

which in a specific form becomes,

$$\text{pressure in C D I K,} = \frac{\textit{length } l \times \textit{pressure in } \text{C D E F}}{\textit{length } l''}$$

$$P''' = \frac{l \times P;}{l'''}$$

which in a specific form becomes,

$$\text{pressure in C D A B} = \frac{\textit{length } l \times \textit{pressure in } \text{C D E F}}{\textit{length } l'''}$$

Now each of these equations expresses the relation that subsists between the ordinates of the common rectangular hyperbola, and the corresponding abscissas when referred to the asymptotes; and, consequently, by viewing the subject in this way the curve of expansion becomes assimilated to the common hyperbola. It therefore appears, that the determination of the pressure of the air at any proposed position of the piston requires the determination of the hyperbolic ordinate at that point, and to effect this, requires the delineation of the hyperbola itself.

There are various ways for delineating the hyperbola when the asymptotes and a point in the curve are given, these being the precise data of the problem which we are now investigating. This coincidence of data is rather a fortunate circumstance in the present inquiry, for by it we are enabled to exhibit to the eye the intensity of pressure at any point, and the approximate effect of expansion throughout any proposed space, for all elastic fluids whatever, when the pressure is given for any known space.

The construction of the hyperbola is effected either by points or by continued motion; the latter method however, implies the possession of an instrument constructed for the purpose, a condition which cannot at all times be complied with, and when the instrument is not at hand, recourse must be had to the method of ordinates as computed from the equation of the curve, or else to some of the well-known mechanical methods, which determine the points in the curve without calculation.

This is by far the readiest and most expeditious method of constructing the hyperbola, and when performed by a skilful operator with correct and delicate instruments, the delineation can be effected with a degree of accuracy far beyond anything that can ever be required in practical operations.

One of the best and readiest methods of finding the points in the curve, when the asymptotes and one point are given, is that which follows:—

Let A D and A C *fig.* 3, be the given asymptotes perpendicular to each other, and let P be the given point in the curve, being such that its position is known with respect to the asymptotes A D and A C; then through the given point P, draw any number of lines at pleasure, as *a* C, *b d*, *c e* and B*f*, to terminate in the asymptotes at C, *d*, *e*, *f* and *a*, *b*, *c* and B; and at these lines take the distances C *g*, *d h*, *e i* and *f k* respectively equal to their counterparts or corresponding distances *a* P, *b* P, *c* P and B P; then will *g*, *h*, *i* and *k* be points in the curve of the hyperbola, *m* P *n*, of which the asymptotes are A D and A C; therefore if through these points the curve line *n g h i k m* be traced, it will be the common hyperbola or curve of expansion sought.

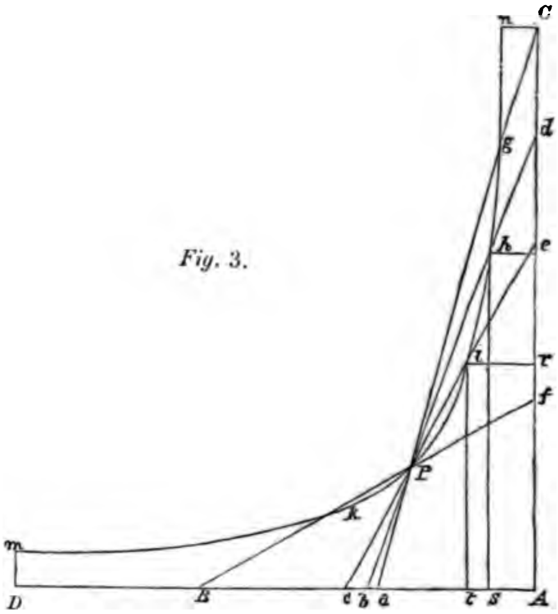


Fig. 3.

Through the points at *h* and *i* anyhow taken in the curve, let the co-ordinates *h e*, *i t* and *h s*, *i r* be drawn respectively parallel to the asymptotes A D and A C; then by the property of the hyperbola, we have

as the abscissa A *r*, is to the ordinate *h e;* so is the abscissa A *e*, to the ordinate *i r;*

and by converting this analogy into its equivalent equation, it assumes the following form:

$$\textit{ordinate } ir = \frac{\textit{abscissa } \mathrm{A}e \times \textit{ordinate } he}{\textit{abscissa } \mathrm{A}r}$$

Now this is the identical equation which we have previously obtained from Mariotte's law of expansion, the rectangle A *e* × *h e* in the numerator, being precisely the same as the constant product *l* × P in the foregoing class of analogies; and moreover, all the rectangles similarly constructed upon their respective co-ordinates, or on the ordinates and abscissas of the hyperbola are equal among themselves.

It therefore appears, that we are perfectly justified in assimilating the common hyperbola with the curve which represents the law of expansion or dilatation in elastic fluids, supposing that the temperature of the fluid remains the same in every state of possible density in which the fluid may be found.

The above is a very simple and elegant method of constructing the hyperbola, when the asymptotes and a point in the curve are given, and as these are the only data supplied for the solution of the problem, no other method of construction need ever be resorted to; and in order to show the application of the hyperbola to the subject in hand, we shall conceive a cylindrical vessel of twelve feet in length, to be one-fourth part full of air of atmospheric density, this air being wholly cut off from all communication with that without the vessel, and confined in that space by means of a piston fitted air-tight, but free to move upwards or downwards, in obedience to the action of any sufficient force, such as the elasticity of the confined air.

Let the air which is above the piston be entirely withdrawn from the vessel, while that which is below it remains of atmospheric density; then, the pressure on the under side of the piston will in the first instance be equal to fifteen pounds upon a square inch of surface, and in consequence of this pressure below, which is altogether unresisted above, the piston will ascend and allow the air to occupy a larger space, the pressure of the air becoming less and less as the space becomes greater, until it is diffused throughout the whole vessel, when the pressure will be only one-fourth part of what it was at first, or 3.75 lbs. per square inch, for by the general law previously referred to, it is,

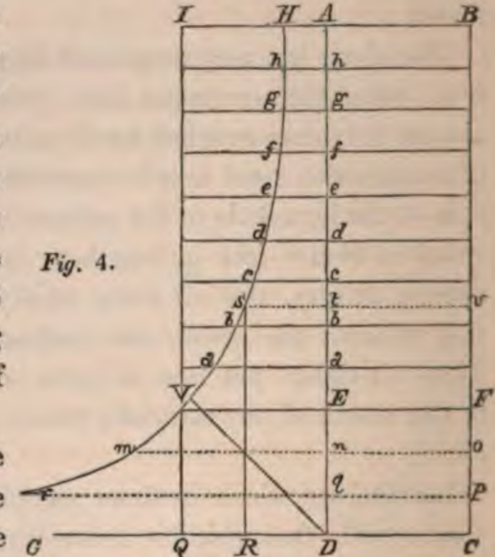
as the whole length of the vessel, is to the whole pressure, so is one-fourth the length, to the pressure sought.

Or, numerically thus, 12 : 15 :: 3 : 3.75 lbs. the pressure per square inch,

when the air is made to occupy the entire vessel; 15 lbs. being the pressure per square inch when the vessel is only one-fourth full, or when the air is in its usual state of density. Now, according to this principle or law, it becomes a matter of the greatest simplicity to represent the force of the air at every point in the length of the vessel during the process of dilatation; for we have only to consider the distances along the length of the vessel estimated from its lowest point, as abscissas of the common hyperbola, and the corresponding ordinates referred to the asymptotes, will show the pressure of the air when it occupies so much of the vessel as the place of the ordinate indicates.

Let A B C D, *fig.* 4, be the proposed cylindrical vessel, twelve feet in length from B to C, and of uniform diameter throughout the whole of its length, the lowermost three feet, or one-fourth of the whole, viz., C D E F being filled with air of atmospheric density, and consequently sustaining a pressure of fifteen pounds upon every square inch of its interior surface, the air being wholly extracted from A B F E the upper portion of the vessel, leaving a vacuum throughout three-fourths of the whole.

Fig. 4.



Produce C D the diameter of the vessel directly foreward to any distance at pleasure; then A D the vertical side of the vessel, and D G the production of the diameter, may be considered as the rectangular asymptotes of the hyperbola, whose ordinates respectively indicate the pressure of the internal air, according to the position which they occupy on the asymptote or vertical side of the vessel proposed.

Produce F E directly foreward, and from some convenient scale of equal parts, set off E P equal to the atmospheric pressure, or fifteen pounds upon a square inch of surface; then will P be a given point in the curve of the hyperbola, of which D A and D G are the rectangular asymptotes. Let the hyperbola therefore be delineated after the manner described above, and take any number of points in the vertical asymptote D A; then if ordinates *a a*, *b b*, *c c*, *d d*, &c., be drawn through these points parallel to D G, these ordinates will represent the pressure of the air on a square inch of the piston, when it has ascended to the several positions which the ordinates occupy, or when the air has expanded itself throughout the corresponding portions of the vessel. Therefore, if these several ordinates be taken in the compasses, and applied to the same scale from which E P was taken, the numerical values of the successive pressures corresponding to the different degrees of expansion, will be very nearly approximated. And if the other portions of the several lines, contained between the curve of the hyperbola and the vertical line P I be similarly measured, they will respectively denote the diminution of pressure at the several positions of the piston. Hence we can at all times determine the intensity of pressure exerted by the air, when the position of the piston is known.

If the same quantity of air of atmospheric density, were compressed into a less space instead of being expanded into a greater, the increase of pressure as propagated by condensation, would be much more rapid than its diminution by expansion; this is obvious from the nature of the hyperbola itself, for if ordinates be drawn at equal distances below the atmospheric line E F, as indicated by the dotted lines *o m* and *p r*, these ordinates will meet the other branch of the hyperbola beyond the vertex at V, and being parallel to the asymptote of that branch, they must extend to a greater distance before they meet the curve in that direction. Let the space C D E F which is filled with air of atmospheric density, be divided into three equal parts, and suppose the piston to be forced successively into the positions represented by *n o* and *q p;* then when it is at *n o,* the force of compression, or the effort of the air to resist further condensation will be $22\frac{1}{2}$ lbs. upon a square inch of surface, for since it is now compressed into two of the parts of which it before occupied three, and because by the general law, the pressures are inversely as the spaces, we have as follows:

2 : 3 : : 15 : 22.5 lbs., the pressure per square inch when the air is compressed into the space *o n* D C, and this pressure is represented by the hyperbolic ordinate *m n*.

Again, when the piston is at *q p*, the force of compression will be 45 lbs. upon the square inch, or equal to three atmospheres; for the space C D *q p* by construction, is only one-third of the original space C D E F, in which the pressure was 15 lbs. upon the square inch, or it is one-half of the space *o n* D C, which contains air of a pressure equal to 22.5 lbs. per square inch; hence it is

1 : 3 : : 15 : 45, or it is 1 : 2 : : 22.5 : 45 lbs. per square inch.

So that by either of these analogies, the pressure in the space *p q* D C is equal to 45 lbs. upon the square inch, which pressure is indicated by twice the ordinate *m n*, or by the ordinate *q r* extended until it meets the curve.

These remarks however, apply only to the case, in which the air is compressed into a less space, than that which it originally occupied when it was of the same density as the atmosphere, for when it is expanded from its primitive density, so as to fill the vessel, if it were again to be compressed into the space which contained it before expansion, the pressures at the several positions of the piston, would be precisely the same as those that took place during the dilatation; but these pressures would display themselves in a reverse order, the elasticity increasing more and more rapidly as the condensation proceeds by the descent of the piston.

Since by reason of its elasticity the air has power to move the piston from one end of the cylinder to the other, commencing with a pressure of 15 lbs. upon a square inch, and terminating with three and three quarter pounds, it is obvious that there must be a mean pressure, which, if allowed to exert itself on the piston throughout the motion, would produce precisely the same effect as is produced by the variable pressures that take place during the expansion of the air in the vessel.

Now, this mean pressure must like all the other pressures, be represented by some ordinate to the asymptote A D, and it must be such, as being multiplied by the corresponding abscissa, shall produce a space equal to D E P Q, for by the property of the hyperbola, all the rectangles under their ordinates and their corresponding abscissas are equal among themselves, and we have elsewhere stated, that the pressure of an elastic fluid, in every state or condition of its density, when multiplied by the space which it occupies, is always of the same value, so that if the mean pressure takes place when the piston is in the position $t\,v$, it will be represented by the ordinate $s\,t$, and the rectangle $s\,t$ D R, will be equal to the rectangle D E P Q, according to the general law of elastic fluids.

The method of determining the mean pressure, requires the application of the differential or fluxional analysis, a method which we will have occasion to notice hereafter; but it may nevertheless be approximated with very great accuracy by means of a rule which mathematicians have investigated for finding the areas of irregular curvilinear spaces; and this rule may be expressed in words as follows:—

> Rule.—*To the sum of the two extreme ordinates, (their number being odd), add four times the sum of the even ordinates and twice the sum of the odd ones, supposing them to be taken at equal distances along the asymptotes; then,*
>
> *Multiply the sum total by the common interval or distance between*

> *any two contiguous ordinates, and one-third of the product will be the area of the space contained between the curve and its asymptote, commencing at the point P, to which add the area of the space D E P Q, and divide the sum by the whole length of the abscissa D A, and the quotient will be the mean ordinate or pressure sought.*

Now, in order to apply the rule to the determination of the mean pressure or ordinate, it becomes necessary to calculate independently, the numerical values of the several pressures or ordinates corresponding to each position of the piston during the process of expansion, taking the first or greatest ordinate at fifteen pounds upon the square inch.

Therefore, by dividing A E the length of that part of the vessel in which the expansion takes place, into intervals of one foot each, the whole length of the vessel being twelve feet, there will be nine intervals contained between ten ordinates or pressures, the first of which has a numerical value of fifteen pounds to the square inch, and the last a numerical value of 3.75 pounds to the square inch; these therefore need not be calculated, but the other ordinates or pressures, eight in number, will be found by the general law as follows:—

4 : 3 : : 15 : 11.25, the pressure or ordinate *a a* in the first interval;
5 : 3 : : 15 : 9.00, the pressure or ordinate *b b* in the second interval;
6 : 3 : : 15 : 7.50, the pressure or ordinate *c c* in the third interval;
7 : 3 : : 15 : 6.43, the pressure or ordinate *dd* in the fourth interval;
8 : 3 : : 15 : 5.63, the pressure or ordinate *e e* in the fifth interval;
9 : 3 : : 15 : 5.00, the pressure or ordinate *ff* in the sixth interval;
10 : 3 : : 15 : 4.50, the pressure or ordinate *gg* in the seventh interval;
11 : 3 : : 15 : 4.09, the pressure or ordinate *hh* in the eighth interval;

Then by taking 15 as the first and 3.75 as the last ordinates or terms of the series, we have according to the foregoing rule, the following operation; viz.:—

$15 + 3.75 = 18.75$, the sum of the extreme ordinates or pressures;

$4(11.25 + 75.0 + 5.63 + 4.50) = 115.52$, four times the sum of the even ordinates or pressures;

$2(9.00 + 6.43 + 5.00 + 4.09) = 49.04$, twice the sum of the odd ordinates or pressures;

Then adding these three results into one sum, we get $18.75 + 115.52 + 49.04 = 183.31$, which being divided by 3, or which is the same thing multiplied by $\frac{1}{3}$, we have $183.31 \times \frac{1}{3} = 61.103$ square feet for the area of the space contained between the curve and its asymptote corresponding to the expansion; to this add the product of 15 by $3 = 45$ the space occupied by the unexpanded air, and we get $61.103 + 45 = 106.103$ square feet for the entire space through which the air acts; consequently, dividing by 12, the whole length of the vessel in feet, we get $106.103 \div 12 = 8.842$ for the mean ordinate or pressure sought.

It is however, proper to remark in this place, that the above method of finding the sum of the series, is not exactly according to the rule, because a condition is therein implied which does not in reality obtain in the present instance; namely, that the number of terms shall be odd; but the error arising from this source is very trifling, for the true mean as determined by the fluxional analysis, is 8.949, whereas by the above approximation, it is 8.842, giving a difference of 0.107 in the length of the mean ordinate.

But if we wish to ascertain the mean pressure in the space through which the air expands, or the upper three-fourths of the vessel, the result by the above rule will err in excess, for it gives 6.789 for the mean ordinate in that space, whereas the fluxional analysis gives only 5.199, being a difference of 1.59 in the mean, which in a practical point of view, may be considered of some importance.

SECTION II.

PROPERTIES OF STEAM IN CONTACT WITH THE LIQUI

WHEN steam or the vapour of water is confined in a close vessel in contact with the liquid from which it is generated, the true theoretic law which shows the relation between the temperature and the elastic force, has not hitherto been elicited, and it is very probable, that a formula which would accurately represent the series of pressures corresponding to the entire range of the scale of temperatures, is beyond the power of analytical science to produce. But if such a formula is actually attainable, it is likely to assume a very complicated aspect, and by involving a high exponent, considerable difficulty may arise in effecting its reduction.

The eminent French mathematician BIOT has proposed an equation, which with very slight deviations from the results of experiments, is alike applicable to every point in the scale of temperature; but then it requires such extensive modifications and alterations, to adapt it to the measures employed in calculating the steam engine, that it becomes almost useless for that purpose, however useful it may be as an instrument of calculation in other researches connected with the properties and effects of steam and other elastic fluids.

We therefore omit the consideration of the formula recorded by the above-named individual, and substitute instead of it several other results which have been obtained with great care, and which, although they do not individually

that temperature being easily obt

rsons who are obliged to take offe
medicine would first take a bit of alum in
mouth, they could then take the medicine
as much ease as though it were so much

apply to the whole range of temperature, yet when taken in connexion, they are sufficient to complete a table on Fahrenheit's scale, ranging from zero to upwards of fifty atmospheres or fifteen hundred inches of mercury, beyond which point it is unnecessary to proceed for any purpose of a practical nature in reference to the steam engine.

The first in order of the formulæ here alluded to, is that of Mr. Southern; it is applicable to pressures less than that of the atmosphere, or fifteen pounds upon the square inch, and consequently to temperatures less than 212 degrees, which is the boiling point of water on Fahrenheit's thermometer. If we express the pressure upon a square inch in pounds avoirdupois, the formula by which it is represented, is

$$pressure = (0.0064216 \times temperature + 0.329426)^{5.13} + 0.04948$$

This equation has sometimes been given in another form; but since its application as here represented, is sufficiently simple, we deem it unnecessary to crowd our pages with different formulæ when no practical advantage can be derived from them.

It may here be proper to remark, that since the exponent of the term involving the temperature is fractional, it cannot be reduced otherwise than by logarithms, it will therefore be advantageous to express the practical rule in terms of these numbers, in as far at least as they become necessary in performing the reduction. The practical rule in words at length, is as follows, viz:—

> RULE.—*Multiply the given temperature in degrees of Fahrenheit's thermometer, by the constant decimal* 0.0064216, *and augment the product by* 0.329426; *multiply the logarithm of the sum by* 5.13; *find the natural number corresponding to the product, to which add the constant* 0.04948, *and the sum will be the pressure in pounds per square inch.*

It is customary with some engineers to estimate the pressure in pounds per circular inch; and when that is the case, the calculation is somewhat simplified in finding the power of a steam engine; for in finding the area of the piston, the multiplication by the fraction 0.7854 is entirely avoided, and the square of the diameter only is taken into the account. Here follows an example for illustrating the above rule.

Example.—Suppose the temperature of the steam to be 176 degrees of Fahrenheit's thermometer; it is required to determine the corresponding pressure in pounds per square inch.

Here by operating as the rule directs, we get

$176 \times 0.0064216 = 1.1302016$

Constant fraction $= 0.3294260$, add

Sum $= 1.4596276$ . . log. 0.1642420

5.13, multiply

Natural number $= 6.95922$ . . log. 0.8425615, product

Constant correction $= 0.04948$, add

Sum $= 7.00870$ pounds per square inch, the pressure sought, being something less than half an atmosphere. The corresponding pressure per circular inch, will be found if the above result be multiplied by .7854; thus it is $7.0087 \times .7854 = 5.5045$ pounds.

The next formula that claims our attention is that of Tredgold, as modified and corrected by Mellett. This carries us upwards from one to four atmospheres, and consequently, it embraces a range of from fifteen to sixty pounds upon the square inch, and admits of a temperature of from 212 to 293 degrees of Fahrenheit; but this range is not sufficiently extensive for modern practice, as steam is now worked at very high pressures for the purpose of more conveniently carrying out the principle of expansion.

The formula as originally given by Mr. Tredgold, is the simplest in its form of any that we have seen applied to this or similar purposes, and this simplicity it owes to the circumstance of its involving nothing but whole numbers; but as altered by M. Mellett, the exponent only is retained, the numbers 100 and 177 being changed into 103 and 201.18 respectively.

The formula thus altered and adapted to logarithmic operation is as follows:—

$$\log. \mathit{pressure} = 6 \times \log. \, (0.0049707 \times \mathit{temp.} + 0.511979);$$

and for the common arithmetical process, it is

$$\mathit{pressure} = \left(\frac{\mathit{temp.} + 103}{201.18}\right)^6$$

The practical rule for the logarithmic operation may be expressed in words at length in the following manner:—

Rule.—*Multiply the given temperature in degrees of Fahrenheit's thermometer, by the constant decimal* 0.0049707, *and increase the product by* 0.511979; *multiply the logarithm of the sum by* 6, *and the natural number answering to the product will be the pressure sought in pounds per square inch.*

Example.—Suppose the temperature of the steam to be 250 degrees of Fahrenheit's thermometer; what is the corresponding pressure in pounds per square inch?

The operation, when performed according to the above rule, is as follows, viz.:

$$\begin{array}{llll}
 & 250 \times 0.0049707 = 1.242675 & & \\
 & \text{Constant fraction} = 0.511979, \text{ add} & & \\
\hline
 & \text{Sum} = 1.754654 \;\;.\;\;. & \text{log. } 0.2441915 & \\
 & & \quad\quad\quad\quad 6 & \text{multiply} \\
\hline
\text{Natural number} = 29.184 \text{ pounds} \;\;.\;\;. & & 1.4651490 & \text{product}
\end{array}$$

The practical rule for determining the pressure by the common operations of arithmetic, is as follows:—

Rule.—*Increase the given temperature of the steam by the constant number* 103; *divide the sum by* 201.18, *and raise the quotient to the sixth power or degree for the pressure sought in pounds per square inch.*

The process by this method will of course be exceedingly tedious, especially if there happen to be many decimal figures arising from the division by 201.18; but the labour of calculation may be avoided by using logarithms according to the following directions:—

Rule.—*To the given temperature of the steam, add the constant increment* 103; *then, from the logarithm of the sum subtract the logarithm of the constant denominator* 201,18, *and multiply the remainder by* 6 *for the logarithm of the pressure sought.*

The above example wrought out by this rule is as follows:—

$$\begin{array}{lll}
250 + 103 = 353 \;\;.\;\;.\;\;.\;\;.\;\;.\;\;. & \text{log. } 2.5477747 & \\
\text{Denominator} = 201.18 \;\;.\;\;.\;\;.\;\;. & \text{log. } 2.3035848 & \text{subtract} \\
\hline
 & \quad\;\; 0.2441899 & \\
 & \quad\quad\quad\quad 6 & \text{multiply} \\
\hline
\text{Natural number } 29.184 \text{ pounds} \;\;.\;\;. & \quad\;\; 1.4651394 & \text{product}
\end{array}$$

The *Comte de* Pambour gives another set of constants for the same range of temperature, in which the exponent 6 is retained, as being the most likely to be the true one, in consequence of its simple form, and when the formula is represented logarithmically, it is as follows, viz.:—

$$\text{log. } pressure = 6 \times \text{log. } (0.0050362 \times temp. + 0.497608),$$

and the practical rule for reducing this equation, is precisely the same as the preceding, with the exception of the constant quantities, which of course must be changed when this equation is employed in any particular calculation. The result of the foregoing example is deduced from this form of the expression, is 29.383 pounds upon the square inch, differing from the former 0.199 part of a pound. The operation is as under :—

$$
\begin{array}{llr}
250 \times 0.0050362 = 1.259050 & & \\
\text{Constant fraction} = 0.497608 & & \\
\hline
\text{Sum} = 1.756658 \quad . \quad . & \text{log. } 0.2446873 & \\
 & \qquad\qquad 6 & \text{multiply} \\
\hline
\text{Natural number} = 29.383 \text{ pounds} & \text{log. } 1.4681238 & \text{product}
\end{array}
$$

By the application of Southern and Tredgold's formulæ, we are enabled to compute a table of pressures from zero up to four atmospheres, or sixty pounds upon a square inch of surface ; but since this extent is not sufficient to embrace every case for which such calculations are required, it becomes a matter of importance to possess a rule, by which the pressures corresponding to higher temperatures may be ascertained.

M M. Arago and Dulong have been successful in their researches in reference to the properties and effects of steam when generated under very high pressures, and they have supplied us with a formula of very easy application for calculating the pressures from four up to fifty atmospheres, and consequently embracing a range of from 60 to 750 lbs. pressure upon the square inch; beyond this we need not proceed, as it is amply sufficient for every purpose connected with the steam engine as at present constructed.

The following is the expression for the rule above alluded to ; it is precisely similar in form to those already given, differing only in the exponent and the other constants, and for this reason, the practical rule which it supplies will be similarly expressed.

$$pressure = (0.0067585 \times temp. + 0.26793)^5$$

In consequence of the high power to which the expression requires to be involved, and the complex decimal numbers that enter its composition, it becomes necessary to have recourse to logarithms for the purpose of facilitating the process ; but in the first place, we must adapt the expression to this mode of operation, it is as follows :—

$$\text{log. } pressure = 5 \times \text{log. } (0.0067585 \times temp. + 0.26793).$$

And the practical rule for reducing the expression, when given in words at length, may be as underneath.

> Rule.—*Multiply the given temperature in degrees of Fahrenheit's thermometer, by the constant co-efficient* 0.0067585, *and augment the product by the fraction* 0.26793 *as a correction; find the logarithm answering to the sum; multiply this logarithm by the given exponent* 5, *and the natural number answering to the product will be the pressure per square inch in pounds.*

Since the formula from which this rule is deduced, applies to higher temperatures than those previously illustrated, it requires an independent example to show the mode of its reduction, such as that which follows.

Example.—Supposing the steam to be generated under a temperature of 308 degrees of Fahrenheit's thermometer, what is the corresponding pressure in pounds per square inch?

The operation by the above rule is as below.

$308 \times 0.0067585 = 2.081616$
Constant fraction $= 0.267930$

Sum $= 2.349546$ . . log. 0.3709839
5 multiply

Natural number $= 71.601$ pounds log. 1.8549195 product

If we compare the several results obtained by the foregoing calculations, with others that have been deduced from experiments with the same, or nearly the same temperatures, the agreement will be found sufficiently satisfactory, and confidence in the correctness and utility of the rules will thereby be established; and since the range is wide enough for every practical purpose, nothing farther on this head can be required.

By means of the foregoing formulæ and the rules deduced from them, we are now enabled to construct a table of pressures, which shall represent the degrees of temperature on Fahrenheit's scale, extending from one pound to 750 lbs. to the square inch, or from one pound up to fifty atmospheres; taking particular care however, to apply each rule to that part of the scale for which it is adapted, according to the exponent and constants therein involved. The four formulæ from which the calculations have been made, when brought into juxta-position with one another, are as follow.

Mr. Southern . log. $pressure = 5.13 \times \log. (0.0064216 \times temp. + 0.329426)$.
M. Mellett . . log. $pressure = 6 \times \log. (0.0049707 \times temp. + 0.511979)$.
De Pambour . log. $pressure = 6 \times \log. (0.0050362 \times temp. + 0.497608)$.
Arago & Dulong log. $pressure = 5 \times \log. (0.0067585 \times temp. + 0.26793)$.

The rules which we have exemplified above are adapted to the determination of the pressure in pounds per square inch, when the temperature is given; but in numerous cases when making calculations in reference to the properties of steam, it becomes necessary to perform the reverse process, and to determine the temperature when the pressure is given.

This however is easily accomplished when the method of transforming algebraic equations is known; but as we write for the instruction of practical men, and not for mathematicians, we shall take the liberty of presenting the equations in a transposed form, and at the same time convert them into practical rules, having the same specific character as those already given for the pressure in terms of the temperature. The first in order for this purpose, is that derived from the transposition of Mr. Southern's formula; it is as below, viz.:—

$$temperature = 155.7256\ (pressure - 0.04948)^{\frac{1}{5.13}} - 51.3$$

The steps of the process necessary for determining the temperature from the pressure, are distinctly indicated by this arrangement; but in order that nothing may be omitted which tends to render the subject intelligible, we here describe the steps of the operation by way of a rule in words.

> Rule.—*From the given pressure in pounds per square inch. subtract the constant correction* 0.04948; *find the logarithm of the difference, and divide it by the exponent* 5.13; *then, to the quotient, add the logarithm of the co-efficient* 155.7256, *and from the natural number answering to the sum, subtract the constant number* 51.3, *and the remainder will be the temperature sought, estimated in degrees of Fahrenheit's thermometer.*

Example.—What is the corresponding temperature to the pressure of 7.00849 pounds upon a square inch of surface, the temperature being estimated in degrees of Fahrenheit's thermometer?

From the above rule we have the following operation, viz.:—

Given pressure in pounds = 7.00849
Constant correction, subtractive = 0.04948

Remainder = 6.95901 . log. 0.8425476

And dividing this by the exponent as directed in the rule, we get 0.8425476 ÷ 5.13 = 0.1642393, to which add the logarithm of the co-efficient 155.7256, and the sum is as below.

$$0.8425476 \div 5.13 = 0.1642393$$
$$\text{Constant} = 155.7256,\ \text{log. } 2.1923601$$

Natural number = 227.3 degrees . log. 2.3565994, sum
Constant decrement = 51.3, subtract

Required temperature = 176 degrees of Fahrenheit's thermometer; which being the same as the temperature assumed in the first example, proves the principle of the rule to be correct.

In like manner, by transforming Tredgold's rule for the pressure, as modified and corrected by M. Mellett, it becomes

$$\textit{temp.} = 201.18 \times \textit{pressure } t - 103.$$

This expression for the temperature is simpler in its form than that deduced from Mr. Southern's formula illustrated above, inasmuch as there is no correction required in the radical term, the sixth root of the given pressure only being sought. The method of reduction may therefore be described by way of a rule in the following words :—

> RULE.—*To one sixth part of the logarithm of the given pressure in pounds per square inch, add the logarithm of the constant co-efficient* 201.18; *find the natural number corresponding to the sum, from which subtract the number* 103, *and the remainder will be the temperature required, reckoned in degrees of Fahrenheit's thermometer.*

Here again, the specified limitations must be attended to, which, as we have already remarked, do in the present instance extend from one to four atmospheres, or from fifteen to sixty pounds pressure upon the square inch ; but in order to verify the rule, it will be proper to take for an example, the pressure as previously computed by the rule to the formula from which the present one is deduced; for it is obvious, that if the operation succeed in reproducing the temperature there assumed, it will be a satisfactory proof that the rule is what it should be, and may therefore be depended upon.

Now, taking the pressure at 29.184 pounds per square inch, as previously computed, the corresponding temperature will be found as follows.

Given pressure = 29.184 pounds ⅙ log. 0.2441899
Constant multiplier = 201.18 . . log. 2.3035848, add

Natural number = 353 degrees . log. 2.5477747, sum
Constant correction, subtractive = 103 degrees

Remainder = 250 degrees of Fahrenheit's thermometer, which corresponding with the assumed temperature in the first instance, proves the rule to be correct.

Again, by transforming the equation of the *Comte de Pambour*, in reference to the temperature, it becomes

$$temp. = 198.562 \times pressure^{\frac{1}{6}} - 98.806$$

Now, by comparing this expression with that immediately preceding, it will be perceived to possess precisely the same form and arrangement, differing only in the value of the constant quantities. Consequently, the rule for reducing it will also be the same, and may therefore be given in words at length as follows:—

> RULE.—*To one-sixth part of the logarithm of the given pressure, expressed in pounds per square inch, add the logarithm of the constant coefficient* 198.462; *find the natural number answering to the sum, from which subtract the constant decrement* 98.806, *and the remainder will be temperature required, reckoned in degrees of Fahrenheit's thermometer.*

Example.—Under what temperature must steam be generated to produce a pressure of 29.383 pounds upon the square inch of surface?

The operation for the solution of this example, when performed as directed by the above rule, is as follows: viz.—

Given pressure = 29.383 . . ⅙ log. 0.2446873
Constant Multiplier = 198.562 . . log. 2.2978962, add

Natural number = 348.806 degrees. log. 2.5425835, sum
Constant correction, subtractive = 98.806 degrees

Remainder = 250 degrees, the temperature required, which being the same as that assumed in the previous case of the problem, proves the rule to be correct.

With respect to the formula of Arago and Dulong, as applied to the higher temperatures, when transformed in the same manner as those which we have previously considered, becomes as below; viz.—

$$\textit{Temperature} = 147.962 \times \text{pressure}^{\frac{1}{5}} - 39.6434.$$

Here again, if we except the exponent and the constant numbers, the equation in its general construction is the same as those which precede it, and consequently the practical rule will also be similarly expressed; it is as below.

> Rule.—*To one-fifth part of the logarithm of the given pressure, expressed in pounds per square inch, add the logarithm of the constant coefficient* 147.962; *find the natural number answering to the sum, from which subtract the constant decrement* 39.6434, *and the remainder will be the temperature required, estimated in degrees of Fahrenheit's scale.*

The formula and rule as here given, apply from four to fifty atmospheres, but as an example of the reduction of the equation by means of the rule, we shall take the pressure of 73.601 pounds as previously computed, which will, as in the other cases, verify the rule by reproducing the temperature from which that pressure was obtained.

Given pressure = 73.601 . . ⅕ log. 0.3709839
Constant multiplier = 147.962 . . log. 2.1701497, add

Natural number = 347.643 degrees . log. 2.5411336, sum
Constant correction subtractive = 39.643 degrees

308 degrees, the temperature sought.

We have been very diffuse and particular in pointing out the method of applying the foregoing formulæ, and the rules derived from them; we have been prompted to this in consequence of the very great importance of these and similar calculations in reference to the steam engine. The several equations for the temperature, when brought into juxta-position, are as underneath; viz.—From

Mr. Southern . . . $\textit{temp.} = 155.7256\,(\textit{pressure} - 0.04948)^{\frac{1}{5.13}} - 51.3.$
M. Mellett . . . $\textit{temp.} = 201.18 \times \textit{pressure}^{\frac{1}{6}} - 103.$
De Pambour . . . $\textit{temp.} = 198.562 \times \textit{pressure}^{\frac{1}{6}} - 98.806.$
MM. Arago and Dulong . $\textit{temp.} = 147.962 \times \textit{pressure}^{\frac{1}{5}} - 39.6434.$

If the pressure in pounds per square inch of surface were the only term by which the elastic force of steam is measured, what we have done in the preceding pages would be amply sufficient to meet the wants of the practical engineer; but since there is another term even more general than that which we have hitherto employed, it becomes necessary to take that into consideration also, in connexion with what we have already done.

What we here allude to is, the custom so prevalent amongst practical men of expressing the elastic force of steam in inches of the mercurial column, thirty inches being reckoned equivalent to the pressure of the atmosphere in its medium state. Consequently, by taking this as a standard, and modifying our previous equations for the pressure, so as to represent the elastic force in inches of mercury, we get in the case of Mr. Southern's formula as below; viz.—

$$\textit{Elastic force} = 0.10059 + (0.0073792 \times \textit{temp.} + 0.378287)^{5.13}$$

Now, since the mode of deriving this equation is somewhat peculiar, it may be instructive to the careful and inquisitive reader to trace the steps of its formation; for if this be well understood in the first place, there can be no difficulty in understanding the corresponding modifications similarly obtained from the other equations, which we shall simply exhibit and apply without reference to the method by which they are derived, presuming that the derivation of one is quite sufficient for understanding that of all the rest.

It is well known that the relative specific gravities of water and mercury in their medium state, are in the proportion of 1000 to 13,600, or as 1 to 13.6; it therefore follows, that 30 cubic inches of mercury will be equal in weight to 408 cubic inches of water; for since the magnitudes or bulks are inversely proportional to the specific gravities, we have,

$$1000 : 13600 :: 30 : 408 \text{ cubic inches of water.}$$

That is, as the specific gravity, or the weight of one cubic foot of water, is to the specific gravity, or the weight of one cubic foot of mercury, so is the weight of 30 cubic inches of mercury to the weight of 408 cubic inches of water.

Now, it has been abundantly confirmed by experiment, that one cubic foot of water, of which the specific is represented by 1000, weighs exactly $62\frac{1}{2}$ pounds avoirdupoise; hence we have, *As the number of cubic inches in one cubic foot is to sixty-two and a half pounds, so is four hundred and eight cubic inches of water to its weight in pounds ; that is,*

$$1728 : 62.5 : : 408 : \frac{2125}{144} = 14.7569 \text{ pounds.}$$

But this, as we have already seen, is equal to the weight of 30 inches of mercury ; and it is a well known fact, that a column of mercury 30 inches in height, standing on a base of one square inch, exerts the same pressure, or is of the same weight as a column of air of the same base and reaching to the top of the atmosphere; we therefore get

$$\frac{2125}{144} : 30 : : p : \frac{30 \times 144 \times p}{2125} = 2.03294 \times p.$$

Where the symbol p denotes any pressure whatever, whether given *a priori*, or computed by rules deduced from some general principle. Therefore, if instead of the symbol p in the above result, we substitute its value as represented in the different equations for the pressure previously obtained, we shall have the elastic force of steam estimated in inches of mercury, and expressed in terms of the temperature and known quantities. Thus, by making the specified substitution in Mr. Southern's equation for the pressure in pounds per square inch, we shall obtain the following form for the elastic force in inches of mercury.

$$\textit{Elastic force} = 2.03294\{0.04948 + (0.0064216 \times \textit{temp.} + 0.329426)^{5.13}\}$$

The equation as it now stands is of too complicated a form for ready reduction; in order, therefore, to simplify it as much as possible, it becomes necessary to multiply each of the terms between the brackets by 2.03294, which gives,

$$\textit{Elastic force} = 0.10059 + 2.03294\ (0.0064216 \times \textit{temp.} + 0.329426)^{5.13}$$

And finally, if each of the terms within the parenthesis be multiplied, the 5.13th root of the coefficient 2.03294, we get,

$$\textit{Elastic force} = 0.10059 + (0.0073792 \times \textit{temp.} + 0.378287)^{5.13}$$

Now this is precisely the expression for the elastic force, which we stated to be derived from Mr. Southern's formula for the pressure in pounds per square inch; and if we compare the equation just obtained with that from which it is derived, it will be perceived that they are perfectly symmetrical in form, and differ from each other only in the numerical values of the constant quantities.

It is therefore obvious, that the practical rule for reducing both equations, will be expressed by the same words, the only difference being in the use of the constant quantities; but since the force of the steam is here expressed in inches of the mercurial column, we think proper to adapt the rule to that term also. It is therefore as follows:—

> RULE.—*Multiply the given temperature of the steam expressed in degrees of Fahrenheit's thermometer, by the constant decimal* 0.0073792, *and increase the product by the constant increment* 0.378287; *find the logarithm of the sum and multiply it by the exponent* 5.13. *Take out the natural number answering to the product, to which add the constant* 0.10059, *and the sum will express the elastic force of steam in inches of the mercurial column.*

Another form of the equation which represents the law of the elastic force in terms of the temperature and constant quantities, is as follows:—

$$\textit{Elastic force} = \left(\frac{\textit{temp.} + 51.26374}{135.5155}\right)^{5.13} + 0.10059.$$

This is even a more elegant form of the equation than that from which the above rule is deduced, and it is obtained by simply casting the fraction 0.0073792, out of both the terms under the exponent 5.13. The practical rule for effecting the reduction is as follows:—

> RULE.—*To the given temperature expressed in degrees of Fahrenheit's thermometer, add the constant increment* 51.26374, *and from the logarithm of the sum, subtract the logarithm of the denominator* 135.5155; *multiply the remainder by the exponent* 5.13, *and find the natural number answering to the product, to which add the constant fraction* 0.10059, *and the sum will be the elastic force of the steam in inches of the mercurial column.*

Example.—Taking, therefore, the temperature of the steam at 176 degrees of Fahrenheit's thermometer, what is the corresponding elastic force in inches of mercury, as calculated by each of the above rules?

By the first rule, the process is as follows:—

Given temperature multiplied by 0.0073792, is equivalent to 176 × 0.0073792 = 1.2987392; then, if to this the constant increment be added, we get

1.2987392 + 0.378287 = 1.6770262 . log. 0.2245409
5.13, multiply
Natural number = 14.18713 . . . . . . log. 1.1518948, product.
Constant correction = 0.10059, add
Sum = 14.28772 = elastic force in inches of mercury.

By the second rule, the process is as follows:—

Given temperature increased by

$51.26374 = 176 + 51.26374 = 227.26374$ log. 2.3565301

Constant denominator $= 135.5155$ log. 2.1319892, subtract

Remainder = 0.2245409

5.13

log. 1.1518948 product.

Here, then, the resulting logarithm being precisely the same as by the first rule, the one becomes a verification of the other; but we would prefer the second mode of working, as it has slightly the advantage in point of facility.

The same result would be obtained by multiplying together the pressure in pounds per square inch, as computed by the rule for that purpose, and the constant fraction $\frac{864}{425}$; indeed, if the pressure in pounds per square inch be merely doubled in any particular case, the result will be sufficiently accurate for every practical purpose, as a pound in pressure per square inch, is generally reckoned equivalent to two inches of mercury, and it is upon that supposition, that the mercurial gauges are constructed.

In practice, the mode of estimating the pressure in pounds per square inch, as being equal to half the number of inches of mercury, cannot lead to any considerable or material error, and since it is a very simple mode of estimation in a rough way, it may be advisable generally to adopt it.

In this case however, the pressure of the atmosphere is supposed to be exactly equal to fifteen pounds upon the square inch, and also to thirty inches of mercury; whereas by a previous computation, we have found it to be only 14.7569 pounds in its medium state. The rough estimate however, will in most cases be sufficiently near the truth, for stating the quality of the vacuum in the working of a condensing steam engine when operating on the principle of expansion.

In like manner, if Mr. Tredgold's formula for the pressure in pounds per square inch, as altered and corrected by M. Mellett, be modified for the purpose of expressing the elastic force in inches of mercury, corresponding to any degree of temperature within the limits of its range, it becomes as follows, viz.

$$\textit{elastic force} = \frac{864}{425}\left(\frac{\textit{temp.} + 103}{201.18}\right)^{6};$$

which expression is reducible to the following, viz.

$$\textit{elastic force} = (0.0055946\ \textit{temp.} + 0.576244)^6.$$

This expression is precisely similar in form to that for the pressure within the same limits, differing only in the numerical value of the constant quantities, and in the terms by which the elastic force of the steam is expressed; and when adapted to logarithmic operation it is as under, viz.

$$\text{log. } \textit{elastic force} = 6 \times \text{log. } (0.0055946 \times \textit{temp.} + 0.576244)$$

And the practical rule for reducing it according to this last form, is as follows.

> Rule.—*Multiply the given temperature in degrees of Fahrenheit's thermometer by the constant decimal* 0.0055946, *and increase the product by the constant increment* 0.576244; *then multiply the logarithm of the sum by the exponent* 6, *and the natural number answering to the product, will be the elastic force of the steam, expressed in inches of the mercurial column.*

Example.—Taking the temperature of the steam at 250 degrees of Fahrenheit's thermometer; what is the corresponding elastic force of inches of mercury?

Here according to the rule, the given temperature multiplied by the constant 0.0055946, gives

$$\begin{array}{rrl}
250 \times 0.0055946 = & 1.398650 & \\
\text{Constant correction} = & \underline{0.576244}, & \text{add} \\
\text{Sum} = & 1.974894 & \text{. . . log. } 0.2955439 \\
 & & \qquad\qquad\quad \underline{6} \\
\text{Natural number} = & 59.3284 \text{ inches} & \text{. log. } 1.7732634
\end{array}$$

The equation for the elastic force of steam in inches of the mercurial column, according to this portion of the scale of temperature, is sometimes expressed otherwise; but it is needless to multiply formulæ and rules for effecting the same end, when one of a sufficiently simple form is enough for every practical purpose.

Since the equation given by the *Comte de Pambour* for the pressures, refers to the same portion of the scale of temperatures, as that form which the preceding rule is derived, it is needless to modify it for expressing the elastic force in inches of mercury; but the equation of Arago and Dulong taking in a wider range, it becomes necessary to adapt it for expressing the elastic force in inches of the mercurial column. When thus modified, it becomes

$$\textit{elastic force} = (0.0077889 \times \textit{temp.} + 0.308778)^5;$$

and this equation when adapted to a logarithmic process, is as follows :—

$$\log.\ \textit{elastic force} = 5 \times \log.\ (0.0077889 \times \textit{temp.} + 0.308778).$$

Here again, the equation assumes a form symmetrical with that for the pressure in pounds, as referred to the same portion of the scale of temperatures; the only difference being in the value of the constant quantities, and the practical rule for reducing the equation is as follows, viz. :—

> Rule.—*Multiply the given temperature by the constant fraction* 0.0077889, *and to the product add the constant fractional correction* 0.308778; *then, multiply the logarithm of the sum by exponent* 5, *and the natural number answering to the sum in the table of logarithms, will be the elastic force in inches of the mercurial column.*

Example.—Taking the temperature of steam at 308 degrees of Fahrenheit's thermometer, what is the corresponding elastic force in inches of mercury?

Here by the rule, the given temperature multiplied by its constant coefficient, is

$308 \times 0.0077889 = 2.398981$

Constant correction $= 0.308778$, add as directed

Sum $= 2.707759$ . . . . log. 0.4326101
5

Natural number $= 145.56$ inches of mercury . log. 2.1630505

Now, by the rule previously given for the pressure in pounds per square inch, we found the pressure corresponding to the temperature 308 degrees, to be 71.601 pounds; therefore, if this be multiplied by the general fraction $\frac{864}{425}$, we get

$71.601 \times 864 \div 425 = 145.56$, the same as above.

It may therefore be taken as a general rule, to whatever portion of the scale of temperatures the calculation refers, that when the pressure is expressed in pounds per square inch, it may be converted into inches of mercury, by using the constant multiplier 2.03294, or when very great accuracy is not required, the number 2 is quite sufficient for every practical purpose.

On numerous occasions, when estimating the effects of steam in the process of expansion, it becomes necessary to calculate the temperature corresponding to a given pressure or elastic force. Now this has already been done by the rules adapted to that purpose, for the several ranges of temperature as specified in the foregoing pages, and when the pressure is expressed in pounds per square inch; and it is obvious that a similar set of rules may be established for the same purpose, when the elastic force of the steam is expressed in inches of mercury; but since the derivation and illustration of these equations would lead us too far into detail, while at the same time it might be offering more than is necessary in a practical point of view, we shall for this reason, content ourselves with that deduced from Mr. Tredgold's formula as corrected by M. Mellett, comprehending a range of from one to four atmospheres.

When it is necessary to ascend to a higher point in the scale of temperatures, our object will be attained if the formula of Arago and Dulong as applied to the pressure, be transformed in a similar manner. Mr. Tredgold's corrected formula, when modified for the present purpose, becomes

$$\textit{temp.} = 178.7436 \times \sqrt[6]{\textit{elastic force}} - 103.$$

This equation is very simple in its form, and it moreover possesses the property of being reducible without logarithms, a property which it derives from the exponent $\frac{1}{6}$, which simply indicates that the square root of the cube root of the term or quantity to which it is applied has to be extracted; but this is much more readily and simply done by logarithms.

The formula however, in its present form, is not well adapted for logarithmic operation, in consequence of its terms being separated from one another by the minus or negative sign. But since the method of extracting the cube in the usual way is not generally known, we prefer expressing the rule logarithmically, as in all the preceding cases.

> RULE.—*To one-sixth part of the logarithm of the elastic force, expressed in inches of mercury, add the logarithm of the constant co-efficient* 178.7436, *and from the natural number answering to the sum, subtract the constant* 103°, *and the remainder will be the temperature required.*

Example.—Taking the elastic force of steam estimated in parts of the mercurial column, at 59.3287 inches; what is the corresponding temperature in degrees of Fahrenheit's thermometer?

Here by the rule it is,

Given elastic force 59.3287 inches . . ⅙ log. 0.2955442
Constant co-efficient 178.7436 . . . . log. 2.2522305, add

Natural number . 353° . . . . Sum=log. 2.5477747
Constant decrement 103, subtract

Remainder = 250 degrees of Fahrenheit's scale, the temperature required.

It will be borne in mind, that the above formula and the rule derived from it, are only applicable to a range of pressures from one to four atmospheres; but if in the course of practice, a higher range should be required, the formula of Dulong and Arago can easily be adapted to the purpose, for which reason we think it is unnecessary to enlarge farther on this part of the subject.

Another subject of very great importance to be known in the application of steam, is the volume, or the space which it occupies under a given pressure, as compared with that of the water from which it is generated; for without a knowledge of this element, no correct calculation in reference to the performance of a steam engine can be made; and strange as it may appear, there is not a single particular in the whole theory of that stupendous agent, with which practical men in general are so little acquainted.

The law of Mariotte, as announced in our general theory of the pressure of elastic fluids, does not of itself apply to the determination of the volume, or the law of vaporization; but when combined with a kindred property discovered by the celebrated experimentalist Gay Lussac, another relation is obtained, by which we are enabled to represent the variations that take place in the bulk or volume of steam, when a simultaneous change takes place in both the temperature and the pressure. The property here alluded to is simply as follows, viz.:—

> *That if the temperature of a given weight of any elastic fluid, be made to vary, while the tension or elastic force remains the same, it will receive augmentations of volume or bulk, which are directly proportional to the augmentations of temperature.*

Or, in other words, for each degree in the variation of temperature, as referred to Fahrenheit's scale, there takes place a corresponding variation in the volume or bulk under the same intensity of pressure.

Now, it has been abundantly proved by experiment, that the volume of fluid under a given pressure, will, for every degree in the rise of temperature on Fahrenheit's scale, be augmented by the 0.00202th part of its volume at the temperature of 32 degrees, or the temperature of freezing water; therefore,

Let v = the volume occupied by a given weight of an elastic fluid, under a given pressure, and at a temperature of 32 degrees of Fahrenheit's scale;

v' = the volume occupied by the same weight of fluid, under the same pressure and temperature t;

v'' = the volume occupied by the same weight of fluid, under the same pressure and temperature t';

Then is $t-32$, the number of degrees of temperature, corresponding to the augmentation in the volume of the fluid, when it is increased from v to v'; and in like manner, $t'-32$, is the number of degrees of temperature, corresponding to the augmentation of volume from v to v'', so that the term $0.00202\,(t-32)\,v$ denotes the quantity of augmentation in the one case, and $0.00202\,(t'-32)\,v$, the quantity of augmentation in the other; consequently, the augmented volumes, are

$$v' = v + 0.00202\,(t-32)\,v, \text{ and } v'' = v + 0.00202\,(t'-32)\,v, \text{ respectively.}$$

And by converting the equations into a ratio, we obtain the following, viz.:

$$v' : v'' :: 1 + 0.00202\,(t-32) : 1 + 0.00202\,(t'-32);$$

Consequently, by the principles of proportion, we get

$$\frac{v'}{v''} = \frac{1 + 0.00202\,(t-32)}{1 + 0.00202\,(t'-32)}$$

An equation from which the volume of the fluid at the temperature of 32 degrees, has been eliminated in the comparison of the terms.

Now, suppose it were required to determine the volume, or the space occupied by a given weight of steam, in passing from one temperature and pressure to another temperature and pressure. This it will be perceived, requires the combination of Mariotte's law, which we have expounded in the general theory of elastic fluids, with the property announced by Gay Lussac, and recently mentioned in these pages. The result of his combination, is

$$\mathrm{V}=\frac{1700\times 14.7569}{p}\left\{\frac{1+0.00202\,(t-32)}{1+0.00202\times 180}\right\}$$

In which equation, the symbol v denotes the volume of steam from a given bulk of water, p the pressure in pounds per square inch, and t the temperature of the steam, estimated in degrees of Fahrenheit's thermometer.

With respect to the numbers 1700 and 14.7569, the first is the volume from a cubic foot of water, generated at a temperature of 212 degrees of Fahrenheit's scale, and the second is the corresponding pressure in pounds per square inch. When the formula is reduced to its lowest terms, it becomes,

$$\mathit{Volume}=\frac{37.1629}{\mathit{pressure}}\left\{(\mathit{temp.}-32)+495.0496\right\}$$

In order therefore to apply this equation in its present form, it becomes necessary to calculate the pressure in terms of the temperature, or the temperature in terms of the pressure, by the rules already given for that purpose, those rules being applied to that portion of the scale for which they are particularly adapted. The logarithmic form of the above equation is as follows:—

$$\log.\ \mathit{volume}=\log.\ 37.1629+\log.\left\{(\mathit{temp.}-32)+495.0496\right\}-\log.\ \mathit{pressure}$$

This is the particular form of the equation which should be applied in calculation, as it abbreviates the labour very considerably. The practical rule which it supplies, is expressed in words in the following manner,—

> Rule.—*From the given temperature subtract* 32 *degrees, which is the temperature of freezing water; then, to the logarithm of the remainder, when increased by the constant number* 495.0496, *add the logarithm of* 37.1629, *and from the sum subtract the logarithm of the pressure in pounds per square inch, and the natural number answering to the remainder, will be the number of times that volume of steam exceeds that of the water from which it is generated.*

When the given pressure is between one and four atmospheres, it must be calculated by the rule which is adapted to that particular range; and when it is between four and fifty atmospheres, the rule of M.M. Arago and Dulong must be employed.

Example.—Suppose the temperature of steam to be 259 degrees of Fahrenheit's thermometer; what is its volume from a cubic foot of water, when generated under a pressure due to that temperature?

In order to resolve this question, we must first of all determine the pressure due to the given temperature, which by the rule for that purpose, is as follows:—

Given temperature=259°
Given correction, additive=103
Sum=362 . . . log. 2.5587086
Constant denominator=201.18 . . log. 2.3035848, subtract
Remainder=log. 0.2551238
6
Natural number=33.9424 . . . . . log. 1.5307428

Having thus determined the pressure due to the given temperature, we must now proceed as directed by the foregoing rule, from which we get as follows,

$$\left.\begin{array}{lll} 259-32+495.0496=722.0496 \; . \; . \; . \; . & & \log.\ 2.8585671 \\ \text{Constant co-efficient}=\ 37.1629 \; . \; . \; . \; . & & \log.\ 1.5701091 \\ \text{Computed pressure}=\ 33.9424 \; . \; . & \text{ar. co.} & \log.\ 8.4692572 \end{array}\right\}, \text{ add}$$

Natural number, 790.56 cubic feet, the volume sought. log. 2.8979334, sum.

Again, if the temperature of the steam be taken at 325 degrees of Fahrenheit's scale, what will be its volume from a cubic foot of water, when generated under a pressure due to the given temperature?

Since the proposed temperature corresponds to a pressure greater than four atmospheres, the value of the pressure must be determined by the rule adapted to that particular portion of the scale, and from which we have as follows,

$325 \times 0.0067585 = 2.196513$
Constant increment$=0.267930$
Sum$=2.464443$ . . . . log. 0.3917165
5
Natural number, 90.904 pounds per square inch . . . log. 1.9585825

Then, according to the rule for the volume, we must proceed as follows:—

$$\left.\begin{array}{lll} 325-32+495.0496=788.0496 & \ldots\ldots & \text{log. } 2.8965536 \\ \text{Constant co-efficient}=37.1629 & \ldots\ldots & \text{log. } 1.5701091 \\ \text{Computed pressure}=90.904 & \ldots \text{ ar. co.} & \text{log. } 8.0414175 \end{array}\right\}, \text{ add}$$

Natural number, 322.16 cubic feet, the volume sought log. 2.5080802, sum.

The Comte De Pambour's volume for a pressure of 91 pounds upon a square inch, 322.7 cubic feet, and for a temperature of 325 degrees, it is 310 cubic feet, with a pressure of 94 pounds upon the square inch.

The reason of the difference, is to be sought for in the circumstance of his using 14.706 pounds per square inch for the pressure of the atmosphere, whereas in our formula we employ 14.7569 pounds; but the difference itself is of no consequence in practice, and the result obtained by our formula is probably the most correct.

In cases where the expansive principle is brought into operation, it is obvious that none of the formulæ hitherto employed can be expected to give correct results, in consequence of the variation that takes place in the pressure of the steam in going through the engine, but nevertheless, rules have been obtained by observation and experiment, which, although strictly speaking, are empirical, yet they give results sufficiently near the truth, and have besides the advantage of giving the pressure and volume in terms of each other.

One form of the equation for the volume in terms of the pressure and known quantities, is as below, viz.:

$$\log. \textit{volume} = \log. 30193.24 - \log. (\textit{pressure} + 4.29046).$$

Now this equation being wholly free from exponental terms, is very easily reduced; but since it is exhibited in terms of the pressure in pounds per square inch, it is not so convenient for our immediate purpose, unless the pressure be directly specified, which is not always the case in this sort of inquiry.

The expression may however, be represented in terms of the temperature, by merely substituting for the pressure in this equation, its particular value in terms of the temperature according to that portion of the scale to which the calculation is referred, a condition determinable by the proposed temperature itself. The practical rule for reducing the foregoing equation in terms of the pressure, is as follows.

> Rule.—*To the given pressure in pounds per square inch, add the constant increment* 4.29046 ; *then subtract the logarithm of the sum from the logarithm of the constant number* 30193.24, *and the natural number corresponding to the remainder, will be the volume of steam in cubic feet, generated from a cubic foot of water.*

Example.—Suppose the steam to be generated under a pressure of 45 pounds per square inch, what will be the corresponding volume from a cubic foot of water ? Or in other words, how many times the space occupied by the water from which it is generated, will steam of 45 pounds pressure upon a square inch occupy ?

Given constant or numerator, 30193.24 . . .	log. 4.4799097
Given pressure increased by 4.29046=45+4.29046=49.29046 . . .	log. 1.6927628 subtract
Natural number, 612.56 cubic feet, the volume sought	log. 2.7871469 remainder.

Hence it appears, that the steam which is generated from a cubic foot of water under a pressure of 45 pounds on every square inch of surface, will fill a vessel of 612.56 cubic feet capacity. Pambour's Table gives 608 cubic feet for a pressure of 45 pounds. The formula for the volume in terms of the temperature and known quantities, is as represented below, viz.

$$\log.\,\textit{volume} = \log.30193.24 - \log.\left\{(0.0049707 \times \textit{temp.} + 0.511979)^{6} + 4.29064\right\}.$$

Where it will be observed that the parenthetical term within the brackets, is the same as the formula for calculating the pressure in terms of the temperature given in another part of the work.

When the pressure is greater than four atmospheres, or above 60 pounds pressure per square inch, the volume of steam corresponding to a given temperature, is expressed by the following equation, viz.

$$\log.\,\textit{volume} = \log.\,30193.24 - \log.\left\{(0.0067585 \times \textit{temp.} + 0.26793)^{5} + 4.29064\right\}.$$

It is not necessary to give rules and examples for the purpose of reducing and illustrating these formulæ, for when the parenthetical term in each of them has been computed by the rule for that purpose, the remainder of the operation is performed as directed in the rule immediately preceding, to which it is only necessary to refer.

The element or volume which we have just shown the method of computing, is of the utmost importance in estimating the effects of a steam engine, and ought to be very attentively studied by practical engineers. It finds its application in assigning the velocity with which steam, generated under a given pressure, flows into a vacuum, and that therefore, is the next object that claims attention.

It is a well-known law in the doctrine of pneumatics, that a free elastic fluid under a given pressure, flows into a perfect vacuum with the same velocity that would be acquired by the power of gravity acting through the height of a uniform column of the fluid capable by its weight of producing the proposed pressure on one square inch of its base. The solution of the present problem therefore reduces itself to the determination of the height of the uniform column here mentioned, and the formula by which this height is represented in feet, is as under,

$$\textit{height of column} = 69565.36 \times \textit{pressure} \div (\textit{pressure} + 4.29046),$$

and this equation, when adapted to logarithms, becomes

$$\log.\ \textit{height of column} = \log.\ \textit{pressure} + \log.\ 69565.36 - \log.\ (\textit{pressure} + 4.29046)$$

When the pressure is given in pounds per square inch, the practical rule deduced from the logarithmic form of the equation, is as follows.

> Rule.—*To the logarithm of the given pressure in pounds per square inch, add the logarithm of the constant co-efficient* 69565.36, *and from the sum subtract the logarithm of the pressure when increased by the constant correction* 4.29046, *and the natural number answering to the remainder, will be the height of the uniform column required in feet,* and from this column, the velocity with which steam rushes into a vacuum is readily found.

The above equation, as well as that which precedes it, being perfectly free from surd or other exponental expressions, may be resolved directly by the rules of common arithmetic, when the pressure or the elastic force is given *a' priori*, or can be assigned from the conditions of the question without calculation; but in order to carry out our system of arrangement, we think proper to express the practical rule in each case logarithmically, and in the present instance, the formula adapted to this purpose is as below, viz.

$$\log.\ \textit{height of column} = \log.\ \textit{elastic force} + \log.\ 69565.36 - \log.\ (\textit{elastic force} + 8.72224)$$

Now the practical rule for finding the height of the column in feet, when expressed logarithmically, is as follows, viz.

> Rule.—*To the logarithm of the given elastic force expressed in inches of mercury, add the logarithm of the constant co-efficient or multiplier* 69565.36, *and from the sum subtract the logarithm of the elastic force when it has been augmented by the constant increment* 8.72224, *and the natural number answering to the remainder, will be the height of the uniform column required.*

Example.—Suppose the elastic force of steam to be equal to 91 inches of the mercurial column; what will be the height of a uniform column of the same steam, to produce the same effect by means of its own weight?

Given elastic force, 91 inches .	log. 1.9590414
Constant multiplier, 69565.36 .	log. 4.8423931, add
Sum =	log. 6.8014345
Given elastic force increased by 8.72224 = 99.72224	log. 1.9987921, subtract
Natural number = 63480.8 feet, the height of the column	log. 4.8026424, remainder

This result it will be perceived, is very nearly the same as that obtained by a previous calculation in terms of the pressure, and it would have been precisely the same, if the assumed elastic force had been such as to correspond to the pressure there employed; as it is however, the coincidence is merely accidental, and implies nothing more than this, that the assumed pressure in the one case, and the elastic force in the other, are such as nearly to correspond in the scale of temperature.

We are now prepared to calculate the velocity with which steam of any given pressure, or any given elastic force or temperature, will flow into a vacuum, on the supposition that it is perfectly free and unconstrained, either by contractions in the passages, or any other cause of resistance whatever, tending to diminish the velocity of influx.

Example.—Taking the pressure of the steam at 45 pounds upon a square inch of the containing surface; what height of a column of uniform density, will suffice by means of its own weight, to produce an equal pressure on one square inch of its base?

Given pressure, 45 pounds per square inch . log. 1.6532125
Constant co-efficient or multiplier, 69565.36 . log. 4.8423931, add

Sum=log. 6.4956056
Given pressure increased by the constant 4.29046=45 + 4.29046=49.29046 . log. 1.6927628, subtract

Natural number, 63510 feet, the height of the column required . . . log. 4.8028428, remainder

It therefore appears, that for steam of a pressure of 45 pounds upon a square inch, it will require a column of 63510 feet in height standing upon a base of one square inch, to produce an equal pressure by means of its own weight; and the temperature of this steam will be somewhere about 274 or 275 degrees of Fahrenheit's thermometer. The height here required is something more than twelve miles, and more than double the height of a uniform column of atmospheric air.

The foregoing equation, for calculating the height of a uniform column of steam of a given weight, is represented in terms of the pressure in pounds per square inch; but since this is not the only form in which the conditions of the problem may be proposed, it may be as well to show the corresponding form of the equation, when the elastic force of the steam is expressed in inches of mercury, of which 2.03294 cubic inches are equal in weight to one pound.

Now we have already shown, that the elastic force of the steam in inches of mercury, is equal to the pressure in pounds per square inch, multiplied by the constant number 2.03294; that is,

elastic force=2.03294 times the *pressure* ;

and from this we obtain the pressure equal to the elastic force divided by the constant 2.03294 ; that is,

pressure=*elastic force* ÷ 2.03294

Let this value of the pressure be substituted instead of it, in the foregoing equation for the height of the column, and that height will then be expressed in terms of the elastic force, as follows, viz.

$$\textit{height of column} = 69565.36 \times \textit{elastic force} \div (\textit{elastic force} + 8.72224)$$

And if we wish to express the height of the uniform column in terms of the temperature, we must make a corresponding substitution to the above, in terms of that element.

Now, it is a well-known law in mechanical science, as referred to the free and unobstructed descent of bodies by their own weight, that if

$g = 32\frac{1}{6}$ feet, denote the power of gravity, or the velocity acquired in one second of time by a free descent;

$h =$ the height in feet from which the body falls to acquire the velocity v; and

$v =$ the velocity in feet per second, acquired by the body in falling through the height h.

Then, the general relation which theory and the laws of nature combined, have established among these quantities, is that,

the *square of the velocity*=twice the *power of gravity* × *height* fallen through.

And by extracting the square root of both sides of this equation, we obtain the following value of the velocity acquired by falling through any proposed height; viz.

$$\textit{velocity in feet per second} = 8.0208 \times \sqrt{\textit{height in feet}}$$

Now, if instead of the height in this value of the velocity, we substitute is equivalent as found on a former page, in terms of the pressure in pounds per square inch, we shall have the following equation for the velocity in feet per second, with which steam of a given pressure will flow into a perfect vacuum, viz.

$$\textit{velocity in feet per second} = 8.0208 \sqrt{\frac{69565.36 \times \textit{pressure}}{\textit{pressure} + 4.29046}}.$$

And this, when reduced to its simplest value and accommodated to logarithmic operation, is as follows, viz.

$$\text{log. } \textit{velocity in feet per second} = \text{log. } 2115.508 + \tfrac{1}{2}\{\text{log. } \textit{pressure} - \text{log. } (\textit{pressure} + 4.29046)\}$$

The equation thus modified for the velocity into a vacuum in terms of the pressure, supplies the following practical rule.

> RULE.—*To the given pressure in pounds per square inch, add the constant correction* 4.29046; *then from the logarithm of the given pressure, subtract the logarithm of this sum, and to one half the remainder, add the logarithm of the constant co-efficient* 2115.508, *and the natural number answering to the sum, will be the velocity required in feet per second.*

Example.—Suppose the pressure of the steam to be 45 pounds upon the square inch; with what velocity would steam of that pressure flow into a perfect vacuum, admitting that there is no reduction of velocity in passing through the aperture?

Given pressure per square inch, 45 pounds . log. 1.6532125
Given pressure plus 4.29046 = 49.29046 . . log. 1.6927628, subtract
Remainder = log. 9.9604497,
$\sqrt{\text{Remainder}}$ = log. 9.9802248 $\Big\}$, add
Constant co-efficient or multiplier, 2115.508 . log. 3.3254145
Natural number, 2021.34 feet per second, velocity sought . . . . . . log. 3.3056393, sum

Now, with respect to the theorem for finding the velocity of influx in terms of the elastic force, it is easy to perceive, that it would be of precisely the same form as that which we have given above in terms of the pressure, having given the elastic force instead of the pressure, and the constant 8.72224 instead of 4.29046; it is therefore needless to give a separate formula and rule for this case, since they would be of the same form and expression as those which we have given above, with the exception of the changes which we have just mentioned.

The preceding computation however, determines the velocity that would be obtained by the direct effect of the column that produces it, without considering the diminution that must necessarily take place in passing the aperture. It is therefore obvious, that the velocity thus computed, is greater than can ever take place in a steam engine; but since the quantity of the reduction must necessarily depend upon the nature of the aperture, the real co-efficient must be determined from circumstances peculiar to the case under consideration, and for this reason, we cannot by any previous investigation, so modify the constant quantity as to render it generally applicable.

What we have done in the foregoing pages, has reference to the properties and effects of steam, when confined in a close vessel in contact with the liquid that produces it.—Subject to this condition, it is evident that whatever change takes place in the temperature, must necessarily be accompanied with a corresponding change in the tension or elastic force; because from the very nature of the change effected in the conversion of water into steam, these two elements, namely, the temperature and the elastic force, must be dependent upon each other; and since the density of an elastic fluid is influenced solely by its temperature and pressure, that density must remain invariable when the temperature and elasticity are invariable.

Hence it is, that certain relations exist between the temperature, pressure, and density of steam when in contact with the generating liquid, which relations we have hitherto been endeavouring to establish, and render subservient to the purposes of calculation, as referred to the changes that take place in steam and other elastic fluids under given circumstances.

But when the steam is separated from the liquid which produces it, the relation to which we have just alluded, no longer exists; for under this condition, the one of them may vary without materially affecting the other; that is, the temperature may become greater or less, while the elasticity undergoes no change; and on the other hand, the elasticity may vary while the temperature remains the same. In either case, the density partakes of changes that are known, and is regulated by laws that have been rigorously determined by experiment and theoretical investigation.

SECTION III.

EXPANSION OF STEAM WHEN SEPARATED FROM THE LIQUID.

WHEN steam is cut off from the generating liquid and allowed to expand itself in a separate vessel, its elasticity and density depend upon the same laws, and are modified by precisely the same causes as the air, which we have just discussed; this experiment has shown to be the case within limits embracing a very wide range and admitting the principle of continuity, it must be universally as here stated.

We have already briefly enunciated the general law of elastic fluids, as promulgated by Boyle, and confirmed by Mariotte; but we think proper to repeat it here in a more explicit and comprehensive form, as follows.

GENERAL LAW.—*If the volume or bulk of a given weight of steam or any other elastic fluid, be made to change its magnitude, without at the same time changing its temperature, the tension or elasticity of the fluid, will vary inversely as the space which it is made to occupy.*

By this we mean, that when the magnitude of the fluid becomes greater, its tension or elasticity becomes less; and on the other hand, when the magnitude or volume becomes less, the tension or elasticity becomes greater.

This is a very important proposition in the doctrine of elastic fluids, and upon it chiefly will the subsequent theory be made to depend; it will however require some modification in its more extended and delicate appliances, and these modifications will be introduced when we arrive at that point of the general investigation, where the numerical results require to be brought out with greater precision. Another general law of elastic fluids of equal importance as the foregoing, is that which was first brought into notice by Gay-Lussac, a French mathematician of very great eminence, and one to whom the physical sciences are much indebted for many of the most important improvements. The law here alluded to is as follows.

GENERAL LAW.—*If, while the tension or elastic force of a given weight of fluid remains unaltered, its temperature is made to vary, it will receive augmentations or diminutions of volume, that are directly proportional to the augmentations or diminutions of temperature.*

It is very evident that neither of the laws above expounded can separately be applied to the consideration of steam when in contact with the liquid from which it is generated, as in that case, very different relations take place from those which we have detailed in the state of separation; yet when these laws are used conjointly, they lead to some very useful and important results, to which however, it is not necessary more particularly to advert in this place.

It is the first of the foregoing laws that directly applies to the subject of our present inquiry, but before we proceed to unfold its principles and discuss it in detail, it becomes necessary to define the several terms peculiar to the disquisition. These terms as we mean to designate them, are, the *initial pressure,* or *elasticity; the terminal pressure* or *elasticity;* the *mean pressure* or *elasticity,* and the *index* or *exponent* of *expansion.*

DEFINITION 1. *The initial pressure or elasticity,* is the entire force or pressure exerted by the steam when it first enters the vessel or cylinder in which it operates, or it is that force which it exerts on every square inch of the containing surface at the instant it begins to expand itself.

DEFINITION 2. *The terminal pressure or elasticity,* is that force or pressure which the steam retains after it has expanded itself so as to fill the vessel in which the expansion takes place.

Definition 3. *The mean pressure or elasticity,* is that force or pressure, with which, should the steam act constantly and uniformly throughout the operation, it would produce precisely the same mechanical effect, as it produces by the variable force evolved during the expansion.

Definition 4. *The index or exponent of expansion,* is the number which indicates how often the steam is rarefied in the vessel or cylinder, or it is the number which expresses how often the space, which the steam occupies under the terminal pressure, contains that which it occupies under the initial pressure.

Having premised these several terms, we proceed in the next place to unfold or develope the principles of expansion in reference to steam alone, as expounded by the first of the foregoing general laws, and for this purpose we employ the following figure.

Let ABCD *fig.* 5, represent a section along the axis of a cylindrical vessel, of which the length is AD or BC, and the diameter AB or CD. Let the lower portion CDEF, be filled with steam received directly from the boiler, and under such a temperature as to exert a given force or pressure on every square inch of the containing surface, then it is obvious, from what we have already advanced in the first general law, that while the temperature of the steam remains unaltered as well as the containing space, the tension or elasticity must also remain unaltered; but if the containing space should change its magnitude, while the weight and quantity of the steam remain the same, the tension or pressure, will by the first general law, vary inversely as the magnitude of the containing space; that is, if the containing space be made to decrease, the pressure of the steam will increase in the same proportion; and on the other hand, if the containing space shall be made to increase, the pressure of the steam will decrease exactly in proportion as the space increases.

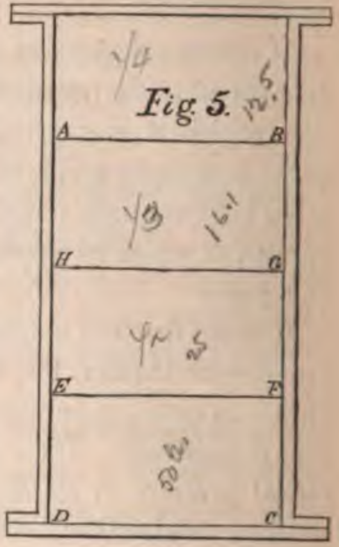


Fig 5.

Thus, let the same quantity or weight of steam which is originally contained in the cylindrical space CDEF, become dilated until it occupies the larger space CDHG; then on the supposition that the temperature remains unaltered or the same in both cases;—

The pressure of the expanded steam, will be to its pressure before expansion as the lesser space CDEF, is to the greater space CDHG.

Now, since the diameter of the vessel is supposed to be uniform throughout the whole of its length, the lengths of any particular parts of it, are accurate representations of the capacities; so that the capacity of the space CDEF, is to the capacity of the space CDGH, as the length DE, is to the length DH. This being the case, it is altogether unnecessary to calculate the capacities by the common rules of mensuration, since the comparison of the pressures can be made by reference to the lengths alone.

We have already obtained an equation for the curve of expansion, with respect to elastic fluids generally, and in order to establish a similar equation for the expansion of steam exclusively,

Put DE $= b$, the length of that portion of the vessel containing unexpanded steam;

EH $= e$, any increase in the length DE, estimated from E along EA;

P $=$ the initial pressure of the steam, or that exerted in the space CDEF;

and $p =$ the pressure when the steam is expanded so as to fill the space CDHG.

Then from what we have said above, combined with the first general law of elastic fluids, the pressure due to the expanded steam, is found by the following analogy.

DH : DE :: P : p; or symbolically thus, $(b+e) : b :: \mathrm{P} : p$.

And if this analogy be reduced by the rules of arithmetic, the equation representing the pressure or force of the expanded steam for any value of e, will be as follows; that is,

$$p = \frac{b \times \mathrm{P}}{(b+e)} \qquad \text{(A)}$$

Say $e = 12$ in

Say $b = 12$ in

$\mathrm{P} = 100$ lbs

in 24) 1200

50 lbs = ½ of P

~~Say~~ $e + b = 24$ in

This equation expresses generally, the tension or elastic force of steam under any proposed degree of expansion, on the supposition that the temperature remains constantly the same, so that the pressure at any particular point of the range of dilatation can always be ascertained, when that at any other point is known previously.

Now, if we suppose the pressures P and p respectively, to represent the ordinates of a curve, having b and $(b+e)$ for their corresponding abscisses;

then the above equation as in the case of atmospheric air, immediately directs us to the common hyperbola, of which the asymptotes and a point in the curve are given, the method of construction having already been exemplified. We have now to apply the general principles which we have here developed, to determine the effect produced by the expansion of steam, when a given quantity or weight of it is confined in a close vessel, such as is capable of enlarging its dimensions in obedience to the pressure which the steam exerts on every point of its interior surface. This is more immediately the subject of inquiry, and in order to render it as simple as possible, we shall suppose the vessel to be in the form of a right cylinder, as this will immediately assimilate the subject to the operation of an expansive steam engine.

Let ABCD *fig.* 6, be a cylindrical vessel of a given length and diameter, and let it in the first place, be filled with steam of a given temperature and pressure to the height EF; then, if the diameter DC be produced directly foreward to X, or to any other distance at pleasure, it is obvious that CB the side of the cylinder and CX the production of its diameter, may be assimilated to the asymptotes of the hyperbola at right angles to each other.

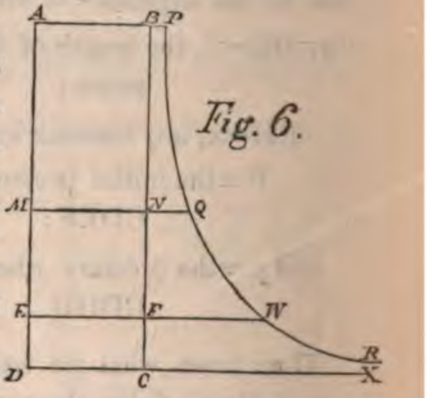


Fig. 6.

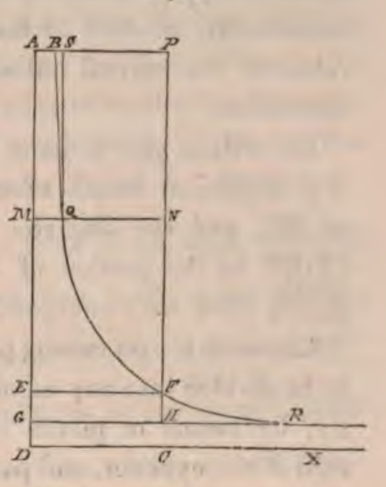
In like manner, if EF be produced to any convenient distance; and if FW be made the linear representative of the force exerted by the steam while it is contained in the space CDEF; then is W a given point in the hyperbolic curve of which the asymptotes are CB and CX.

Consequently, if with these asymptotes and the given point W, the hyperbola PQWR be constructed, an ordinate taken at any point in the asymptote CB, will be an accurate linear representation of the force exerted by the steam, when it has expanded itself so as to fill the vessel as far as the point corresponding to the assumed ordinate.

Thus, if N be any assumed point in the asymptote CB, and if through the point N, the straight line MN be drawn parallel to CD, and produced to meet the curve in Q, then will the ordinate NQ denote the force which confines the steam to the space CDMN, or the force with which it presses on every point of the containing surface, when expanded so as to fill the space below MN; FW representing the initial pressure, or the force which the steam exerts in the space CDEF before the expansion begins; and BP the terminal pressure, or that which the steam exerts after having dilated itself throughout the cylinder.

It will be here observed, that the ordinate BP, bears to the ordinate FW the same proportion that the space CDAB; that is, the ordinates are inversely as the corresponding spaces, a property which holds good at whatever point an ordinate may be assumed in the asymptote to which it is referred.

Fig. 7.



But if we suppose the other side of the cylinder, viz. AD *fig.* 7, and the base DC produced to X, to be the asymptotes of the hyperbola, and F any point in the curve; then if EF the diameter of the cylinder denote the initial pressure of the steam, or the force that confines it in the space CDEF; that part of the curve PQF which follows within the cylinder, will not only exhibit the force which the steam exerts at any point of dilatation, but it will at the same time, exhibit the portion of the force that is lost in consequence of expansion.

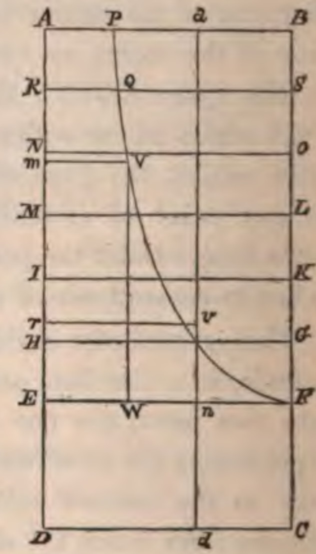
This is perfectly obvious, for the straight line MN drawn across the cylinder in a direction parallel to the base, must be divided by the curve into two parts, the one part, viz. that on the convex side of the curve, representing the force which the steam exerts at that point, and the other part on the concave side of the curve, representing the portion of the original force which the steam has lost in expanding from the space CDEF to the space CDMN. Thus MQ on the convex side of the curve denotes the force that remains in the steam under the given degree of dilatation, and QN on the concave side denotes the portion of the force that is lost.

If on the other hand, it were required to compress the steam into a less space CDGH; then the force which would be necessary to effect the compression and confine the steam in that space, is indicated by the increased ordinate GR, by which we see the very rapid increase of force necessary to compress the steam after it has attained a certain degree of density, and at the same time we may observe how slowly it loses its elasticity after it has attained a certain degree of expansion.

Thus for instance, the force from M to A is almost uniform; the total loss in expanding from CDMN to CDAB being only equal to PS, whereas the loss in expanding from CDEF to CDMN is BS or QN, being about eight times the loss in expanding through very nearly an equal space.

This mode of representing the action of steam during the process of expansion by means of the curve of the hyperbola, is possessed of many advantages, not only as indicating the nature of the expansion and the variation of force, but likewise in leading us to correct conclusions on the subject. Since it happens however, that the ultimate results must always be expressed numerically, we shall in the next place proceed to show how the numerical values of the several ordinates are to be obtained by the usual modes of calculation.

Fig. 8.



Let ABCD *fig.* 8, be a section along the axis of a cylindrical vessel, of which the length is AD or BC, and the diameter AB or CD; and let CDEF be the portion of the vessel which is originally filled with unexpanded steam.

Conceive the remaining part of the vessel ABFE, to be divided into any number of equal parts, say six, by means of planes running parallel to the ends of the cylinder, and passing along the straight lines HG, IK, ML, NO and RS; then, in order to determine the respective spaces occupied by the steam, when by expansion, it has ssuccesively reached to the several planes by which the vessel is supposed to be divided, we must have recourse to the common rules of mensuration; but since the diameter of the cylinder is everywhere the same, the respective spaces are directly proportional to the lengths or distances along the cylinder, and for this reason, it is altogether unnecessary to exhibit the actual capacities, the lengths corresponding to the space being quite sufficient for the purpose.

Denoting by the symbol E, the common interval between the points of division along the cylinder, and by D the constant distance DE; then, the capacities of the spaces corresponding to the positions of the several ordinates, will be strictly proportional to the following expressions, viz.

1. Capacity of the space CDEF, is directly proportional to the length DE;
2. Capacity of the space CDHG, is directly proportional to the length DH;
3. Capacity of the space CDIK, is directly proportional to the length DI;
4. Capacity of the space CDML, is directly proportional to the length DM;
5. Capacity of the space CDNO, is directly proportional to the length DN;
6. Capacity of the space CDRS, is directly proportional to the length DR;
7. Capacity of the space CDAB, is directly proportional to the length DA;

Therefore, by restoring the symbolical representations of the several lengths as specified above, and applying the general law of elastic fluids; then by comparing each of these terms after the first with the first which is constant, we get the following series of analogies for the force of steam in the several spaces corresponding to the several states of rarefaction throughout the whole space ABFE or for the value of the ordinates applied at the several points along the side of the cylinder between E and A. Thus, it is

$DH = (D + E) : D :: P : p = \frac{D \times P}{D+E}$, the pressure or force in the space CDHG;
$DI = (D + 2E) : D :: P : p = \frac{D \times P}{D+2E}$, the pressure or force in the space CDIK;
$DM = (D + 3E) : D :: P : p = \frac{D \times P}{D+3E}$, the pressure or force in the space CDML;
$DN = (D + 4E) : D :: P : p = \frac{D \times P}{D+4E}$, the pressure or force in the space CDNO;
$DR = (D + 5E) : D :: P : p = \frac{D \times P}{D+5E}$, the pressure or force in the space CDRS;
$DA = (D + 6E) : D :: P : p = \frac{D \times P}{D+6E}$, the pressure or force in the space CDAB.

Here it is manifest, that we may continue the series to any extent at pleasure, without actually performing the process for each term separately, the number of terms being wholly dependent on the value given to the interval or symbol E, and when the value of that quantity is indefinitely small, the number of the terms of the series is indefinitely great, and in that case, their aggregate is the true representation of the hyperbolic area contained between the curve and the asymptote, from which the mean pressure is readily obtained.

From the above class of results, it is obvious, that the several pressures, or the ordinates which represent them, are simply expressed in terms of the initial pressure, the length of that portion of the cylinder containing the unexpanded steam and the constant interval between the ordinates. But instead of giving to the symbol E a variable magnitude reckoned from a stated point, we have considered it to be constantly the same, and have repeated it a certain number of times, corresponding to the distance of the point at which any particular ordinate is applied.

This condition is indicated by the numerical co-efficient of E in the denominator of each of the foregoing fractions, and it is attended with very considerable advantage when expressed in this manner, as in consequence of it, we are enabled to calculate the value of any individual ordinate directly with reference to any other terms of the series, the place of the ordinate being always indicated by the co-efficient of the common interval, in that particular term of the series to which it refers.

Although the symbol E, in each of the foregoing pressures or ordinates after the first, is combined with a numeral co-efficient, yet the several terms are perfectly analogous to the equation of the hyperbola as referred to its asymptotes, and consequently they are all of the same import as equation (A), which expresses generally, the value of an ordinate for any abscissa in the series, $(D+E)$, $(D+2E)$, $(D+4E)$, $(D+5E)$, &c., carried on to $(D+nE)$, where the symbol n denotes any number whatever; consequently, if the numerical values of the several pressures or ordinates in the preceding series, be computed from some assumed data, we shall thereby determine as many points in the curve of the hyperbola, from which points the curve may be truly delineated, so as to give the value of every intermediate ordinate throughout the whole length of the cylinder. The following example will render what has here been stated perfectly clear.

Example.—Suppose the whole length of a uniform cylindrical vessel to be 8 feet, and in the first place, let 2 feet of the lower part of it, be filled with steam of such a density, as to exert a pressure of 35 pounds upon every square inch of the interior surface; what will be the pressure at each foot of the remaining part of the cylinder, the steam being allowed to dilate or expand itself gradually throughout the whole length?

In this example it is supposed, that 2 feet of the length of the vessel are occupied with steam of a proposed density, and it is required to find the pressure when the same quantity of steam occupies 3, 4, 5, 6, 7 and 8 feet respectively. Now $D=2$ and $E=1$, the initial value of P, being equal to 35 pounds upon the square inch of the containing surface of that part of the cylinder which is filled with the unexpanded steam; consequently, the several values of the variable pressure, or ordinates representing it, in the foregoing series, will be as follows, viz.

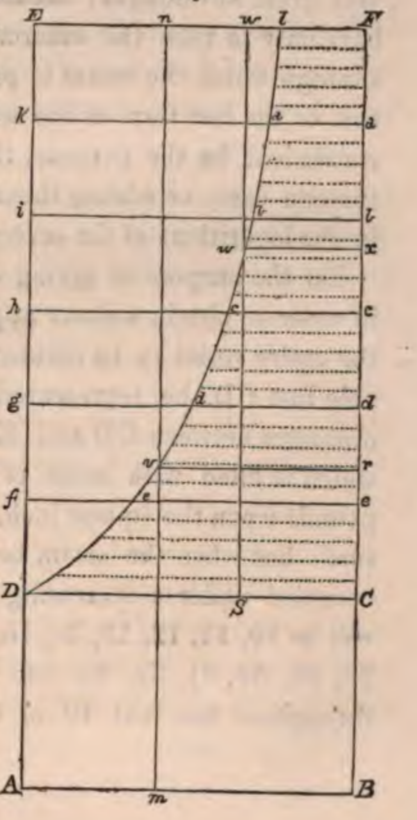
1. Value of p or the pressure at $\mathrm{HG}=\frac{\mathrm{D}\times\mathrm{P}}{\mathrm{D}+\mathrm{E}}=70\div3=23.333$ pounds.
2. Value of p or the pressure at $\mathrm{IK}=\frac{\mathrm{D}\times\mathrm{P}}{\mathrm{D}+2\mathrm{P}}=70\div4=17.500$ pounds.
3. Value of p or the pressure at $\mathrm{ML}=\frac{\mathrm{D}\times\mathrm{P}}{\mathrm{D}+3\mathrm{P}}=70\div5=14.000$ pounds.
4. Value of p or the pressure at $\mathrm{NO}=\frac{\mathrm{D}\times\mathrm{P}}{\mathrm{D}+4\mathrm{P}}=70\div6=11.667$ pounds.
5. Value of p or the pressure at $\mathrm{RS}=\frac{\mathrm{D}\times\mathrm{P}}{\mathrm{D}+5\mathrm{P}}=70\div7=10.000$ pounds.
6. Value of p or the pressure at $\mathrm{AB}=\frac{\mathrm{D}\times\mathrm{P}}{\mathrm{D}+6\mathrm{P}}=70\div8=\ 8.750$ pounds.

Here then, we have the numerical values of six ordinates or pressures computed by the hyperbolic equation already investigated; we may therefore, by combining these computed ordinates with that which expresses the initial pressure of the steam, delineate the curve of expansion for this particular example, which curve will enable us to trace the decrease of force, from the moment that the steam begins to expand itself, until it has filled the entire cylinder.

In order to render the construction of this example as simple and explicit as possible, it will be convenient to consider the diameter of the cylinder itself, as being the ordinate which represents the initial pressure of the steam, and by taking the other ordinates as computed above from the same scale, the exact proportion will obtain at every point of the curve.

Now, if we take CD *fig*. 9, the diameter of the vessel, as the representative of the initial pressure of the steam, or that which it exerts while it occupies the space ABCD, and if from the same scale of equal parts, we set off on the several horizontal lines which divide the cylinder into horizontal segments, the values of the several ordinates or pressures above computed, viz. *ee*, *dd*, *cc*, *bb*, *aa* and F*l*, the curve line *l a b c d e* D passing through the several points to which those values or the respective parallels, will be an hyperbola, having for its asymptotes the length of the cylinder BF, and its diameter BA produced indefinitely towards A.

Fig. 9.



This, as we have already shown by Mariotte's law of the dilatation and pressure of elastic fluids, is the curve which indicates the variation of the pressure or elasticity of steam, when it is allowed to expand itself in a close vessel from some given bulk or volume, where its pressure is known, into some other bulk or volume, or until it occupies the entire vessel.

By contemplating the subject of expansion in this way, it will be divested of much of the mystery and intricacy usually ascribed to it by practical men, and although the result so obtained, may not be rigorously correct when compared with those derived from observation, yet they will be sufficiently so for every practical purpose; and the operative mechanic will thus be enabled to appreciate the advantages to be derived from working steam expansively, and at the same time to approximate to its effects, without having recourse to the refined speculations of abstract theory. If in any particular case, very great accuracy should be required, it becomes necessary when this method of computation is employed, to determine the values of a great number of terms, and this by applying the common rules of arithmetic, would be a very prolix and tedious operation, and in order to facilitate and abridge the process, the aid of logarithms may be called in with very great advantages; because by the employment of these numbers, we have only to take the differences corresponding to the consecutive spaces through which the steam is progressively expanded, and by calculating the first or the last term of the series, according as the one or the other is most convenient for the purpose, then successively subtracting the differences in the one case, or adding them in the other, the remainders or the sums will be the logarithms of the several terms of the series required.

For the purpose of giving an example of this method of computation, and in order to obtain a closer approximation to the true result, we shall suppose the entire vessel to be divided into 40 equal parts, or rather let the moveable line CD, be represented in 30 different positions as indicated by the ordinates between CD and EF; then it is obvious, that the space ABCD, which is filled with steam of the initial density, exerting a pressure of 35 pounds upon the square inch, will contain 10 of those spaces with full pressure; but when the steam begins to expand, the spaces will be successively occupied under a decreasing pressure and the series of spaces so occupied, will be 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39 and 40; but the pressure throughout the first 10 of these equal spaces, is constantly equal to 35 pounds upon the square inch; and when the steam begins to expand, the pressure decreases as the space increases; consequently the series of analogies, will be as under, for the several pressures throughout the process of expansion, taking the constant term at 35 × 10 = 350.

1. 11 : 35 : : 10 : $\frac{350}{11}=31.818$ pounds per square inch.
2. 12 : 35 : : 10 : $\frac{350}{12}=29.157$ „ „
3. 13 : 35 : : 10 : $\frac{350}{13}=26.923$ „ „
4. 14 . 35 : : 10 : $\frac{350}{14}=25.000$ „ „
5. 15 : 35 : : 10 : $\frac{350}{15}=23.333$ „ „
6. 16 : 35 : : 10 : $\frac{350}{16}=21.875$ „ „
7. 17 : 35 : : 10 : $\frac{350}{17}=20.588$ „ „
8. 18 : 35 : : 10 : $\frac{350}{18}=19.444$ „ „
9. 19 : 35 : : 10 : $\frac{350}{19}=18.421$ „ „
10. 20 : 35 : : 10 : $\frac{350}{20}=17.500$ „ „
11. 21 : 35 : : 10 : $\frac{350}{21}=16.667$ „ „
12. 22 : 35 : : 10 : $\frac{350}{22}=15.909$ „ „
13. 23 : 35 : : 10 : $\frac{350}{23}=15.217$ „ „
14. 24 : 35 : : 10 : $\frac{350}{24}=14.583$ „ „
15. 25 : 35 : : 10 : $\frac{350}{25}=14.000$ „ „
16. 26 : 35 : : 10 : $\frac{350}{26}=13.462$ „ „
17. 27 : 35 : : 10 : $\frac{350}{27}=12.963$ „ „
18. 28 : 35 : : 10 : $\frac{350}{28}=12.500$ „ „
19. 29 : 35 : : 10 : $\frac{350}{29}=12.069$ „ „
20. 30 : 35 : : 10 : $\frac{350}{30}=11.667$ „ „
21. 31 : 35 : : 10 : $\frac{350}{31}=11.290$ „ „
22. 32 : 35 : : 10 : $\frac{350}{32}=10.937$ „ „
23. 33 : 35 : : 10 : $\frac{350}{33}=10.606$ „ „

24. $34 : 35 :: 10 : \frac{350}{34} = 10.294$ pounds per square inch.
25. $35 : 35 :: 10 : \frac{350}{35} = 10.000$,, ,,
26. $36 : 35 :: 10 : \frac{350}{36} = 9.722$,, ,,
27. $37 : 35 :: 10 : \frac{350}{37} = 9.459$,, ,,
28. $38 : 35 :: 10 : \frac{350}{38} = 9.211$,, ,,
29. $39 : 35 :: 10 : \frac{350}{39} = 8.974$,, ,,
30. $40 : 35 :: 10 : \frac{350}{40} = 8.750$,, ,,

It cannot be denied, but the appearance of the above calculation is very formidable, but that is the very reason why it has been represented in this form, for by it we mean to show, that in all calculations of this nature, it becomes necessary to abridge the labour as much as possible, either by adopting some artifice for the purpose, or by the employment of logarithms; it is this latter method that we mean to use in the present instance, as being the best adapted for effecting the necessary abbreviation.

It is very evident that since the result in each term is obtained, by dividing the constant term 350 by the several consecutive numbers from 10 upwards to 40, it simply requires the logarithm of each denominator to be subtracted from the logarithm of the constant term, and the several remainders will be the logarithms of the respective ordinates or pressures, corresponding to the spaces which the steam occupies; and the corresponding natural numbers, will express the values of the pressures or ordinates as compared with the initial pressure of 35 pounds upon the square inch.

Now, from what has been said above, it appears that if from the logarithm of the constant term 350, we subtract the logarithm of 11, which is the first term after the expansion begins, the remainder will be the logarithm of the pressure at the first stage of the expansion of the steam from its original bulk.

In like manner, if from the logarithm of the constant term 350, we subtract the logarithms of 12, 13, 14, &c., up to the last term' 40, the remainders will be the logarithms of the series of pressures, corresponding to the several stages or degrees of expansion between the initial and terminal pressures.

But the labour attending this operation may be greatly facilitated by finding the differences of the logarithms 11, 12, 13, 14, &c., up to 40, and subtracting these differences in succession from the first term computed as directed above, and the successive remainders, will be the logarithms of the pressures in the several spaces, diminishing from 35 to 8.75 pounds upon the square inch.

Here follows a table of the logarithmic differences for the example in hand, beginning with 11 and terminating with 40.

NUM.	LOGARITHM	DIFFERENCE
11	1.0413927	377885
12	1.0791812	347622
13	1.1139434	321846
14	1.1461280	299633
15	1.1760913	280287
16	1.2041200	263289
17	1.2304489	248236
18	1.2552725	234811
19	1.2787536	222764
20	1.3010300	211893
21	1.3222193	202034
22	1.3424227	193051
23	1.3617278	184834
24	1.3802112	177288
25	1.3979400	170333
26	1.4149733	163905
27	1.4313638	157942
28	1.4471580	152400
29	1.4623980	147233
30	1.4771213	142404
31	1.4913617	137883
32	1.5051500	133639
33	1.5185139	129650
34	1.5314789	125891
35	1.5440680	122345
36	1.5563025	118992
37	1.5682017	115819
38	1.5797836	112810
39	1.5910646	109954
40	1.6020600	

In the usual mode of calculation, or in any proposed isolated case, the above table may be entirely omitted; it is only shown in this place, to exhibit the series of differences as they succeed each other, but in all cases where the terms are consecutive numbers of their class, having unity for their common difference, the differential quantities above exhibited, may be obtained directly from the table of logarithms without any separate arrangement. The method of using the differences is as follows.

The constant term, 350	log. 2.5440680
First denominator, 11	log. 1.0413927
First term of expansion, 31.82 lbs. .	log. 1.5026753

We shall therefore have the following arrangement for the computation of the entire series from 10 up to 40.

			Logarithms.		Pounds per square inch.
1st	term.	11.	2.5440680 – 1.0413927 = 1.5026753,	nat. num.	31.818
2,	„	12.	1.5026753 – 0.0377885 = 1.4648868,	„	29.157
3,	„	13.	1.4648868 – 0.0347622 = 1.4301246,	„	26.923
4,	„	14.	1.4301246 – 0.0321846 = 1.3979400,	„	25.000
5,	„	15.	1.3979400 – 0.0299633 = 1.3679767,	„	23.333
6,	„	16.	1.3679767 – 0.0280287 = 1.3399480,	„	21.875
7.	„	17.	1.3399480 – 0.0263289 = 1.3136191,	„	20.588
8,	„	18.	1.3136191 – 0.0248236 = 1.2887955,	„	19.444
9,	„	19.	1.2887955 – 0.0234811 = 1.2653144,	„	18.421
10,	„	20.	1.2653144 – 0.0222764 = 1.2430380,	„	17.500
11,	„	21.	1.2430380 – 0.0211893 = 1.2218487,	„	16.667
12,	„	22.	1.2218487 – 0.0202034 = 1.2016453,	„	15.909
13,	„	23.	1.2016453 – 0.0193051 = 1.1823402,	„	15.217
14,	„	24.	1.1823402 – 0.0184834 = 1.1638568,	„	14.583
15,	„	25.	1.1638568 – 0.0177288 = 1.1461280,	„	14.000
16,	„	26.	1.1461280 – 0.0170333 = 1.1290947,	„	13.462
17,	„	27.	1.1290947 – 0.0163905 = 1.1127042,	„	12.963
18,	„	28.	1.1127042 – 0.0157942 = 1.0969100,	„	12.500
19,	„	29.	1.0969100 – 0.0152400 = 1.0816700,	„	12.069
20,	„	30.	1.0816700 – 0.0147233 = 1.0669467,	„	11.667
21,	„	31.	1.0669467 – 0.0142404 = 1.0527063,	„	11.290
22,	„	32.	1.0527063 – 0.0137883 = 1.0389180,	„	10.937
23,	„	33.	1.0389180 – 0.0133639 = 1.0255541,	„	10.606
24,	„	34.	1.0255541 – 0.0129650 = 1.0125891,	„	10.294
25,	„	35.	1.0125891 – 0.0125891 = 1.0000000,	„	10.000
26,	„	36.	1.0000000 – 0.0122345 = 0.9877655,	„	9.722
27,	„	37.	0.9877655 – 0.0118992 = 0.9758663,	„	9.459

		Logarithms.	Pounds per square inch.	
28th term	38.	0.9758663 – 0.0115819 = 0.9642844,	,,	9.211
29, ,,	39.	0.9642844 – 0.0112810 = 0.9530034,	,,	8.974
30, ,,	40.	0.9530034 – 0.0109954 = 0.9420080,	,,	8.750

We have exhibited the arrangement in this way for the convenience of the page, but in the actual computation of any proposed example, the series of logarithmic differences will be subtracted in succession, and the whole operation being thus connected together, the last number computed independently by the theorem which expresses the law of expansion, will be a check on the entire process, so that no error can escape detection by this method, unless it arise in taking out the natural numbers corresponding to the several logarithms, for here, the differences in the results are not sufficiently uniform to indicate where an error occurs; but as those numbers are taken out independently, an error in any one of them will not affect the others. In taking the differences of the logarithms however, the case is far otherwise, for when an error occurs at any point of the operation, it affects all the subsequent steps to the same amount, until it is detected by an independent computation.

If we now collect the several results or pressures as we have just computed them, and divide the sum by their number, the quotient will approximate to the mean pressure as evolved expansion; which mean pressure would, if it continued of the same intensity throughout the operation, produce exactly the same mechanical effect as is produced by the variable pressure evolved during the expansion of steam. And if to the sum of these pressures, we add the effect of the steam in the lower part of the vessel before expansion, and divide the sum by the whole number of intervals or parts for which the pressures are calculated, the quotient will be a near approximate value of the mean pressure both before and after expansion.

Now, the sum of the ordinates in the space where the expansion takes place, is 472.339, and the effect of the unexpanded steam, is 350; the sum of these is 822.339, and the number of intervals is 40; therefore by division, we get

$$mean\ pressure = \frac{822.339}{40} = 20.558 \text{ pounds per square inch.}$$

The correct mean pressure is 20.88 pounds per square inch, which being compared with that above, shows an error of 0.322 of a pound, an error sufficiently small in amount, to show the degree of confidence to be placed in the above mode of computation. But a nearer approximation may be attained, by making use of the rule already specified at page 12, for finding the area of curvilinear figures by means of equidistant ordinates; and for the reader's complete satisfaction, we shall give the result by this method also. The operation is as follows:

Sum of the extreme pressures or ordinates, $35 + 8.750 = 43.750$
Four times the sum of the even pressures, $229.011 \times 4 = 916.044$
Twice the sum of the odd pressures, . $242.328 \times 2 = 484.656$
Sum of these three quantities multiplied by one-third $= 1444.450 \times \frac{1}{3} = 481.483$
Effect produced by the steam in the lower part of the cylinder before expansion . . . . $= 350.000$

Total effect $= 831.483$

and dividing this by 40 the whole number of terms, we get $831.483 \div 40 = 20.787$ pounds per square inch for the mean pressure sought; now this differs from the correct mean by nothing more than the 0.093 part of a pound per square inch, which is too trifling to be taken notice of in practical cases.

The example which we have just illustrated is perfectly intelligible and satisfactory, making allowance for the tedium of the process, and it is besides as simple a case as can be brought under consideration if the steam is allowed to operate by expansion in any degree whatever. This degree of simplicity arises from the circumstance of the steam expanding itself into four times the original space, which gives 4 as the index or exponent of expansion, and the number 4 according to our scale of notation, being one of the simplest for performing the different operations of multiplication, division, involution and evolution.

In order however, to take as general and comprehensive a view of the subject as possible, we shall show the method of obtaining the index of expansion in every variety of cases that can occur, and for this purpose, we shall first conceive the vessel in which the expansion takes place, to be of uniform sectional area; in the same manner as in the cylinder of a steam engine. Then, if the whole length of the vessel be denoted by unity, the index of expansion, or the number which denotes how often the steam is allowed to dilate in the vessel will always be represented by the reciprocal of that fractional part of the length at which the supply of steam from the boiler is intercepted; or which is the same thing, that fractional part of the capacity of the vessel which is occupied with steam of the initial density. Now in every case where steam is allowed to enter a close vessel under a given pressure and in this state to occupy a given portion of it, if the steam be afterwards allowed to dilate itself throughout the whole of the vessel that part of it which was originally filled with steam of a given pressure, will be expressed by a fractional part of the whole, and the difference between the denominator and the numerator of that fractional part, will show the space which the expanded steam is made to occupy; the numerator of the fraction showing the part of the vessel occupied by steam of the original density, and the denominator the whole number of parts into which the vessel is divided. If the denominator of such a fraction be divided by the numerator, the quotient will express the value of the index or exponent of expansion, thereby showing the number of times that the expanded steam occupies its original space, or that in which it was contained before the expansion began.

These things being premised, the mode of approximating to the mean pressure, or the mean elastic force of the steam evolved during the process of expansion, will in every instance be the same as we have stated above, and for this reason, the process need not be again repeated, or even alluded to, but in order to render what has here been said perfectly explicit, we shall exemplify in a few cases, the mode of assigning the value of the index or exponent of expansion, and from it to determine the series of terms for which the pressures have to be calculated.

Let the following fractions respectively, denote the part of the vessel that is filled with steam directly from the boiler; viz. $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{8}$ and $\frac{3}{20}$; then will the series $\frac{7}{8}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{2}$, $\frac{3}{8}$ and $\frac{17}{20}$, denote the number of parts which are filled by expansion; and if the denominators of the first series be respectively divided by the numerators, the quotients will show in each case, the number of times that the steam is expanded. These series under a proper management are as below.

Parts of the vessel filled with steam from the boiler, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{8}$ and $\frac{3}{20}$.
Parts of the vessel filled with the expanded steam, $\frac{7}{8}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{2}$, $\frac{3}{8}$ and $\frac{17}{20}$.
Number of times that the steam is expanded, 8, 4, $2\frac{2}{3}$, 2, $1\frac{3}{5}$ and $6\frac{2}{3}$.

In these examples it will be perceived, that the second or middle series is derived from the first, by simply subtracting the numerators from the denominators, and placing the denominators under the remainders; and in like manner, the third series is derived from the first, by only dividing each denominator by its corresponding numerator.

It will however be observed, that in choosing these examples, no attempt whatever has been made to assimilate them to practical cases generally, thinking it better to select the several fractions at random, the usual instances being so well known to mechanical engineers, that no information would be imparted by discussing them. Since it is the mean pressure of the steam during its action in the cylinder, both before and after it begins to expand, that constitutes the chief object of inquiry, we must endeavour to determine it immediately from the premises, without having to call the aid of any approximating method whatever.

In order to this, we must have recourse to the differential analysis, a method of investigation beyond the reach of the generality of practical and mechanical engineers; but then, it must be understood, that the rule obtained by this method is simple and direct, while at the same time it is perfectly rigorous, if we admit the temperature of the steam to remain constant during the action; for by the general law of elastic fluids:—

The force of the steam is inversely as the space which it occupies.

Now in order to carry out the investigation of the rule here alluded to, it becomes necessary to adopt a certain class of symbols, for the purpose of exhibiting the several steps of the process, a circumstance which, although it confers upon the inquiry something of an abstruse aspect, it is nevertheless indispensable for attaining the desired object. Therefore, put

l=the whole length of the vessel or cylinder considered uniform;
l'=that portion of the length which the steam occupies under full pressure;
n=the index or exponent of expansion, or the number of times that the steam is dilated;
P=the initial pressure of the steam, or that which it exerts before expansion begins;
P'=the terminal pressure, or that which the steam exerts when occupying the whole vessel;
p=the mean pressure of the steam evolved throughout the action;
p'=the mean pressure of the steam evolved under expansion;
x=any increase in the length l', and
dx=an infinitely small increase in the length x.

Then, since the sectional area of the vessel, is considered uniform in figure and magnitude throughout the whole of its length, or as far as the steam acts; it follows, that if the whole length of the vessel be divided by that part of it, which is occupied by the steam is dilated from its original density when it occupies the entire vessel or cylinder.

The quotient thus obtained is what we have called the index of expansion, and if the initial pressure of the steam be divided by this index, the quotient will express the value of the terminal pressure, or that pressure which the steam exerts when it occupies the whole vessel, or when it has expanded to the extent of its range. Thus, we have

$$n=\frac{l,}{l'} \text{ and } \mathrm{P}=\frac{\mathrm{P}}{n}=\frac{l\mathrm{P}}{l'}$$

So that P, P′ and n are mutually dependent on each other, a condition which has a considerable tendency to simplify the theory, by limiting the pressures according to the nature and terms of the construction.

By referring to the preceding notation, if we suppose the length l' to become $l+x$; then by the general law of expansion, the pressure in the space $l'+x$ will, by reason of the dilatation become reduced in the degree indicated by the following proportion, viz.

$$l+x : l' :: \mathrm{P} : p'=\frac{l'\mathrm{P},}{l'+x} \text{the pressure in the space } l'+x,$$

and if the expression be multiplied by the differential dx, we get $\frac{l\mathrm{P}\,dx}{l'+x}$, for the power in expanding through the infinitely small space dx.

Taking the integral of this expression, we get $l\mathrm{P}$ hyp. log. $(l'+x)\,\mathrm{C}$; and when x vanishes, we get

$$l\mathrm{P} \text{ hyp. log. } (l'+x)+\mathrm{C}=o\,;$$

and from this we obtain the complete integral as below, viz.

$$l'\,\mathrm{P} \text{ hyp. log. } \left(\frac{l+x}{l'}\right).$$

This is the expression for the additional power which is gained by expansion, and when $l'+x=l$, we obtain the equivalent value

$$l'\,\mathrm{P} \text{ hyp. log. } \frac{l}{l'}$$

But the power developed by the steam when acting through the space l at full pressure, is expressed by $l'\,\mathrm{P}$; therefore the total power developed during the action, is

$$l\,p=l'\,\mathrm{P}+l'\,\mathrm{P}, \text{ hyp. log. } \frac{l}{l'}=l'\,\mathrm{P}\ (1+\text{hyp. log. } \frac{l}{l'})$$

and dividing both sides of the equation by l, we get

$$p = \frac{l' \mathrm{P}}{l} (1 + \text{hyp. log. } \frac{l}{l'}$$

or since $n = \frac{l}{l'}$ as we have elsewhere shown, it is

$$p = \frac{\mathrm{P}}{n} (1 + \text{hyp. log. } n) \qquad \text{(B).}$$

This is the general form which the equation assumes, on the supposition that MARIOTTE's law of expansion and pressure is correct, and the same expression exhibited in a specific form becomes

$$\textit{mean pressure} = \textit{terminal pressure} \times (1 + \text{hyp. log. } n.) \qquad \text{(C).}$$

Now, this equation gives the correct mean pressure of the steam during the action, always admitting the correctness of the above law, which for all practical purposes is perfectly admissible, as no appreciable error can arise from adopting it.

The expression is as simple in form and arrangement as can be desired, and the practical rule by which it is reduced, may be expressed in words as follows.

> RULE.—*To the hyperbolic logarithm of the index of expansion add unity; multiply the sum by the terminal pressure of the expanded steam, and the product will be the total mean pressure evolved during the operation, or the mean pressure exerted by the steam from the time it begins until it ceases to act upon the piston.*

To show the advantage to be obtained by the application of this rule, we shall take the data of the preceding example, in which the mean pressure was obtained by the computation of thirty ordinates, together with a very tedious summation and other subsidiary processes, which are here altogether dispensed with.

We have already shown that the terminal pressure of the steam is equal to the initial pressure divided by the index of expansion; hence we have

$35 \div 4 = 8.75$ lbs. per square inch for the terminal pressure.

Now, the hyperbolic logarithm of 4, is 1.3862943, to which add unity and it becomes 2.3862943; therefore by the rule it is

$$\textit{total mean pressure} = 2.3862943 \times 8.75 = 20.879 \text{ lbs. per square inch.}$$

In this example the arithmetical mean between the initial and terminal pressures is $\frac{1}{2}(35+8.75)=21.875$ lbs. differing from the true mean by something less than one pound pressure upon the square inch, which shows that even this method of calculating the mean pressure may sometimes be resorted to, especially when great accuracy is not required; but because the same degree of coincidence will not obtain in every case, this mode of calculation is not to be recommended, and ought to be very sparingly adopted in important instances since its application is very little easier or shorter than the correct method, as may be seen by comparing it with the preceding process.

We have now to show that the total mean produce of the steam as thus obtained, is competent to produce the same effect, supposing it to act uniformly throughout the stroke, as is produced by the force evolved by expansion, together with that developed before expansion begins. By the conditions of the question, one-fourth part of the motion is performed by the steam under full pressure, and three-fourths of it by expansion, so that the separate effects under the two conditions will be as follows.

Effect under full pressure, $35 \times 2 = 70$ lbs., the momentum,
Effect by expansion, $12.13 \times 8 = 97.04$ lbs, the momentum,

Sum $= 167.04$ lbs., the total momentum.

The total mean pressure as computed above is 20.879 lbs., and the space through which it acts is 8 feet; hence we get $20.879 \times 8 = 167.622$ lbs. mean momentum, being the same very nearly as that obtained above.

This agreement is very satisfactory, as it proves the result of calculation to be correct by both methods, and shows that the one or the other may be adopted as circumstances may dictate. By referring to *fig.* 9, we may readily determine the point in the hyperbolic curve, at which the ordinate representing the mean pressure ought to be applied; for we have only to divide the diameter EF in *n*, in the proportion of 35 to 20.879 and a line drawn through *n* parallel to the asymptote BF will intersect the curve in *v* the point of the ordinate, and *v r* drawn parallel to the other asymptote AB, will be the ordinate by which the mean pressure is represented.

The point *r* in the asymptote to which the mean ordinate is applied may however, be readily found by calculation, for we have only to transform the general equation (A), in such a manner, as to disengage the symbol *e*, which in the present instance, is the quantity required to be found, since by setting off its value along the asymptote from C, the point of application at r will be ascertained.

By the general equation marked (B), we found the total mean pressure throughout the action to be 20.879 pounds per square inch; hence it is,

$$\frac{10 \times 35}{10 + e} = 20.879, \text{ by equation (A).}$$

And if this equation be reduced in reference to the symbol e, the point of the ordinate will be found as follows, viz.

$$e = \frac{350 - 208.79}{20.879} = 676 \text{ divisions from the point in the asymptote C.}$$

Then from C set off C r along the asymptote CF, equal to 1.352 feet or 6.76 divisions as represented in the figure, and the ordinate $r\,v$ drawn parallel to the other asymptote AB, will be the mean pressure sought.

If unity be omitted from the parenthetical term in the equation marked (B) the remainder will be the form of the expression for the mean force of the steam as developed by expansion alone, exclusive of the force exerted by the steam under full pressure; thus we have

[$P' = \frac{P}{n}$ (hyp. log. n), which reduced to a specific form, becomes

. . *mean force by expansion* = *terminal force* × hyp. log. n . . (D)

It was by this equation that the quantity 12.13 was computed, and the practical rule for its reduction is as follows:—

> Rule.—*Let the terminal pressure of the steam be ascertained, by finding how often the index of expansion is contained in the initial pressure; then multiply the hyperbolic logarithm of the index of expansion by the terminal pressure obtained as above described, and the product will be the mean pressure developed by expansion alone.*

Suppose the initial force of the steam as it enters the vessel to be 35 pounds per square inch as in the foregoing example; what is the additional power gained by expansion, on the supposition that the steam is rarefied four times; that is, dilated into four times the space which it occupied before the expansion began.

The terminal pressure of the steam after expanding throughout the vessel, being equal to the initial pressure, divided by the index of expansion, we get

$$\textit{terminal pressure} = \textit{initial pressure} \div 4 = 8.75 \text{ lbs. per inch.}$$

The hyperbolic logarithm of 4 by the tables, is 1.3862943, which being multiplied by the terminal pressure, gives

$$\textit{mean force by expansion} = 1.3862943 \times 8.75 = 12.13 \text{ pounds per inch.}$$

We can therefore ascertain the point in the asymptote CF, where the mean ordinate must be applied; for since the diameter EF *fig.* 9, is made the representative of the initial pressure, which by the example is 35 pounds per square inch, we have only to divide the diameter EF into parts in the point *w*, which shall be to each other in the ratio of 22.87 to 12.13; then a parallel to the asymptote FC through the point *w*, will intersect the curve in the point *u*, and this point is the place of the o rdinate sought; and if the perpendicular *u x* be drawn from *u*, upon FC, it will be the ordinate representing the mean pressure developed by the steam in dilating itself into four times the original space.

Having thus graphically determined the mean force of the steam as developed by expansion, the position of the ordinate which represents it, may likewise be found by the transposition of equation (A), after the manner which we have shown above for the total mean pressure as developed both before and after expansion. Thus we have

$$\frac{35 \times 10}{10 + e} = 12.13 \text{ lbs. per square inch, by equation (A);}$$

which being reduced in reference to the symbol *e*, gives

$$e = \frac{350 - 12.13}{12.13} = 18.85 \text{ divisions from C, as indicated by the dotted ordinates.}$$

Therefore, by setting off C *x* equal to 18.85, or equal to 3.77 feet along the side of the cylinder from C, and drawing the ordinate *u x* as before, the thing required will be obtained.

The following is the practical rule for this particular case.

> RULE.—*Multiply the difference between the initial pressure and the mean pressure developed by expansion, by the length of that portion of the vessel in which the steam acts with full force, and divide the product by the mean force developed by expansion, and the quotient will point out the position of the ordinate by which the mean pressure is represented.*

By this rule we can always ascertain to what degree of expansion we may proceed, in order to attain any proposed degree of pressure within the proper limits; this however is not precisely the object which the problem demands, for notwithstanding the very great importance of the subject which we have been discussing, in leading to a general and comprehensive view of the doctrine of expansion, yet it has reference only to the additional power which is gained by the dilatation of the steam, and not to the mean force exerted throughout the operation, which indeed is the grand desideratum in computing the actual power of an expansive engine.

If we attentively examine the composition of the general equation marked (B), which indicates the mean pressure developed by the steam during the operation, we find it to involve three distinct quantities, combined with a fourth quantity which is constant, and of the three variable quantities, any two being given, the third can always be found. We have already resolved the equation in reference to the mean pressure, when the initial pressure and the index of expansion are given; we have therefore in the next place, to consider it in regard to the initial pressure, on the supposition that the mean pressure and the index of expansion are known. For this purpose, let both sides of the general equation marked (B), be multiplied by n the index of expansion, and we obtain for the first step of reduction as follows, viz.

$$n \times p = \mathrm{P} \times (\text{hyp. log. } n + 1).$$

Then, if both sides of the equation be divided by the parenthetical member (hyp. log. $n+1$), the expression for the value of the initial pressure becomes

$$\mathrm{P} = np \div (\text{hyp. log. } n + 1);$$

and by reducing this equation to a specific form, we get

$$\textit{initial pressure} = n \times \textit{mean pressure} \div (\text{hyp. log. } n + 1). \quad . \quad . \quad \text{(D)}$$

By this equation, we are enabled to determine under what pressure the steam must enter the vessel, so as to produce a certain mean force under a given degree of expansion, and the practical rule for reducing it, may be expressed in words as follows.

> RULE.—*Multiply the proposed mean pressure of the steam by the number which indicates how often it is expanded from its original bulk, and divide the product by the hyperbolic logarithm of the same number increased by unity, and the resulting quotient will give the initial pressure in pounds per square inch.*

The information here given is of much greater importance in the management of steam than what might at first view be imagined, for by it we are enabled to determine at what pressure the steam ought to be generated in the boiler, so as deducting a given loss, the power remaining may be competent to produce a given effect under given conditions. The following example will elucidate the use of the rule and the equation from which it is derived.

Example.—The total mean pressure of the steam; that is, the pressure exerted both before and after expansion is 20.879 pounds per square inch, and it expands itself into four times the space through which it acts under full pressure; under what pressure does the steam enter the vessel, and what pressure does it exert when fully expanded ?

Here by the rule we have

Total mean pressure $\times n = 20.879 \times 4 = 83.516$ the dividend.

Hyperbolic logarithm of 4 increased by unity, is $1.3862943 + 1 = 2.3862943$, the divisor; hence we have,

initial pressure $= 83.516 \div 2.3862943 = 35$ pounds per square inch very nearly, and from this the terminal pressure, or that which the steam is capable of exerting, after it has diffused itself through four times the space which it is proposed to occupy at full pressure, is 8.75 pounds per square inch, for we have already stated, that the terminal force, is equal to the initial force divided by the index of expansion, so that by division, we get $35 \div 4 = 8.75$ lbs.

From the above rule and the example which we have resolved by it, we perceive how easy it is to ascertain under what degree of pressure, the steam must enter the vessel from the boiler, so as to produce a proposed mean force, after it has expanded a certain number of times. This is indeed a very important consideration in the management of the steam-engine, and supposing the boiler to be of sufficient power for generating steam to embrace the whole range of practice, the principle here illustrated may be applied with very great advantages.

There is, however, another form under which the same equation may be represented, and as that also may be useful on many occasions, we consider it proper in this place to supply it. Now, it is a well-known principle in mathematical science, that if the common logarithm of any number or quantity, be multiplied by 2.3025851, the product is the hyperbolic logarithm of the same number or quantity; consequently by retaining the foregoing symbols, and introducing this constant, we get

$$np = \mathrm{P} \times (2.3025851 \text{ log. } n + 1);$$

and if all the terms of this equation be divided by 2.3025851, it becomes

$$0.4342945 \times np = \mathrm{P} \text{ (log. } n + 04342945).$$

Therefore, in order to reduce this equation in reference to P, we must divide both sides of the parenthetical number (log. $n + 0.4342945$), and we get

$$\mathrm{P} = 0.4342945\ np \div (\text{log. } n + 0.4342945).$$

This is a form very well adapted for logarithmic operation, in consequence of the manner in which the terms are respectively combined, and because the process is much facilitated in this way, we shall modify the expression for the application of these numbers. It is, when so modified, as follows.

$$\text{log. P} = \text{log. } 0.4342945 + \text{log. } n + \text{log. } p - \text{log. } (0.4342945 + \text{log. } n \quad . \quad \text{(E)}$$

Now this equation for the initial pressure is perfectly intelligible and simple, and the practical rule for reducing it may be expressed in words as follows.

> Rule.—*Add into one sum, the logarithm of the constant co-efficient* 0.4342945; *the logarithm of the total mean pressure of the steam, and the logarithm of the index of expansion; then, from the sum of these three logarithms, subtract the logarithm of the number which is made up of the constant* 0.4342945 *added to the logarithm of the index of expansion, and the remainder will be the logarithm of the initial pressure sought.*

Taking therefore the data of the foregoing example, the operation for the initial pressure will be as follows, viz.

Constant co-efficient, 0.4342945	log. 9.6377843
Mean pressure, 20.879	log. 1.3197097
Index of expansion, 4	log. 0.6020600
Sum of the logs. =	1.5595540
The divisor = 0.6020600 + 0.4342943 = 1.0363545 log.	0.0155086
Initial pressure, or natural number, 35 lbs. . . log.	1.5440454

There is still another case of the general problem to be resolved, in order to show, that all the quantities are deducible from the same equation, viz., that which involves the three terms already specified, including the initial pressure, the mean pressure, and the index of expansion.

The methods of finding the mean and initial pressures have already been exemplified, and it now only remains to show how the index of expansion is to be determined, and for this purpose, we must again refer to the general equation marked (B), from which by reduction, we get

$$\frac{1}{n}(\text{hyp. log. } n+1)=\frac{p}{\text{P}};$$

or by converting the expression into common logarithms, it is

$$\frac{1}{n}\ (0.4342945+\text{log. } n)\ 0.4342945\ p\div \text{P};$$

and by throwing the expression into a specific form, it becomes

$$\frac{1}{\textit{index of expansion}}\times\left\{0.4342945+\text{log. } \textit{index of expansion}\right\}=\frac{0.4342945\times \textit{mean pressure}}{\textit{initial pressure}}. \quad \text{(F)}$$

Since this equation assumes a form that can only be reduced by one or other of the methods of approximation, or as it is sometimes called, trial and error, it would in this instance be difficult so to express the equation in in words as to render it intelligible to the general reader. However, a little practice in the method of reduction, will soon render the operator familiar with the process, since it simply consists in assuming such a value for *n*, that its logarithm increased by 0.4342945 shall be equal to a given quantity.

Retaining therefore the same data as before; that is, where the initial pressure is 35 pounds upon the square inch, and the total mean pressure 20.879 pounds; how often is the steam rarefied when these pressures are found to have place?

Here, by a little consideration, we readily perceive, that the value of *n* or the index of expansion, cannot be less than three, nor greater than five; let us therefore assume these numbers, and operate with them as indicated by the equation; we shall consequently, in the first place, by the assumption of $n=3$, have as follows, viz.

$\frac{1}{3}$ (log. $3+0.4342945) = \frac{1}{3}\ (0.4771213+0.4342945) = 0.3038$, nearly; but the known side of the equation, or that which contains nothing but given quantities, when reduced to its numerical value is 0.2591; from which we infer, that the assumed number is too small, for we have

$$0.3038-0.2591=0.0447.$$

Again, if we assume the value of n to be equal to 5, the resulting number obtained as above will be

$$\tfrac{1}{5}(\text{log. } 5+0.4342945)=\tfrac{1}{5}(0.6989700+0.4342945)=0.2267.$$

In this case therefore, the resulting number is less than the given number, from which we infer, that the assumed number is too great, and the error is

$$0.2267-0.2591=0.0324.$$

Consequently, by the first, which is less than the truth, the resulting quantity is more than enough; and by the second assumption, which is greater than the truth, the resulting quantity errs in defect. Now, this is precisely the contrary of what might at first view be expected, but the apparent paradox will readily be answered, if we consider, that the logarithm of a number, divided by the number itself, is continually less and less as the number itself is greater, a circumstance that becomes obvious by examining the table. It therefore follows, that whenever the resulting quantity in our equation, is less than the given quantity, the assumed number from which the deficient result is obtained, is greater than the truth, from which we know in what direction the subsequent assumptions are to be made.

Consequently, by making use of the above two assumptions, and their respective errors, we have as follows,

First assumption, and second error, $3\times 0.0324=0.0972$, product.
Second assumption, and first error, $5\times 0.0447=0.2235$, product.

Sum of the products $=0.0972+0.2235=0.3207$, the dividend; and the sum of the errors, is, $0.0447+0.0324=0.0771$, the divisor; hence we have

$$n=0.3207\div 0.0771=4, \text{ the index sought.}$$

By carrying out the foregoing division, it will be found that 4 is not the exact quotient, the process giving 4.16 very nearly; but by making trial with 4.16 according to the formula, we get

$$\tfrac{1}{4.16}\,(\log.\ 4.16+0.4342945)=\tfrac{1}{4.16}\,(0.6190933+0.4342945)=0.2532\,;$$

a result which is still less than the given quantity, but much nearer to it than the result arising from the assumption of 5. The error in this is only -0.0059 in excess, and being of the same kind with the error arising from the assumption of 5, and of a contrary kind to that arising from the assumption of 3; the corrected result may be obtained from a comparison of 4.16, and its error -0.0059, with either of the former, as we shall show by the following operation:

When 5 is the assumed number, it is $5\times 0.0059=0.0295$, product.
When 4.16 is the assumed number, it is, $4.16\times 0.0324=0.1347$, product.

Since in this case, the errors are both of a kind, the difference of the products must be divided by the difference of the errors, which gives

$$n=\frac{0.1347-0.0295}{0.0324-0.0059}=\frac{0.1052}{0.0265}=3.969$$

Here then we are certain that the real value of n lies between 4.16 and 3.969, and by comparing 4.16 and error, with 3 and its error, we get

When 4.16 is the assumed number, $4.16\times 0.0447=0.1859$, product,
When 3 is the assumed number, $3\times 0.0059=0.0167$, product.

Because in this case, the errors are of different kinds, the sum of the products must be divided by the sum of the errors; whence we get

$$n=\frac{0.1859+0.0167}{0.0447+0.0059}=\frac{0.2026}{0.0506}=4,\ \text{the true value.}$$

Consequently, if the value of n be substituted in the formula, it will be found to succeed; it therefore appears, with how little trouble the true value of the index of expansion can be calculated; for the second correction will in almost every instance, be found sufficiently correct for every practical purpose, provided always, that the numbers are not assumed at a great distance from each other, not very wide of the truth. There is however, another method of calculating the value of n different from that which we have just exemplified, and equally easy of application; for if we calculate the mean pressure by equation (B), for each of the two assumed values of n, and compare the means thus calculated with that which is given, will obtain two errors as before, either alike or unlike, and by working with those errors according to their affections, an approximate result will be obtained as before, and by repeating the process, the result may be carried out to any degree of accuracy at pleasure.

Let us therefore assume the numbers 3 and 5 as before, the initial and mean pressure being 35 and 20.879 pounds upon the square inch respectively; then operating according to equation (B), we have, when 3 is the assumed number, as follows:

$p=\frac{35}{3}(\text{hyp. log. } 3+1)=\frac{35}{3}(1.0986123+1)=24.484$ pounds per square inch.

But the given pressure is 20.879 pounds per square inch, which we know for the reasons already stated to be produced by a higher rate of expansion. Consequently, working by 5 as the assumed value of n, we get

$p=\frac{35}{5}(\text{hyp. log. } 5+1)=7(1.6094379+1)=18.266$ pounds per square inch.

Now, this being less than the given mean pressure, we infer from it, that it arises from a more extensive dilatation than that corresponding to the proposed mean of 20.879 pounds; hence it is evident that the required value of n is situated between the numbers 3 and 5 as assumed above.

When 3 is the assumed value of n, the error is, 3.605, in excess.
When 5 is the assumed value of n, the error is, 2.613, in defect.

Therefore, by proceeding with the assumed numbers and their respective errors as in the preceding case, we have the following process:

First assumption, and second error, $3\times 2.613=7.839$, product.
Second assumption, and first error, $5\times 3.605=18.025$, product.

The sum of the products is, $7.839+18.025=25.864$; and the sum of the errors is, $2.613+3.505=6.218$: therefore, dividing the former by the latter, we get

$n=\frac{25.864}{6.218}=4.16$, the first approximate value; which being precisely the same as we obtained in the preceding case; we may therefore conclude, that by a similar repetition of the process, the final result would be the same also; it is therefore unnecessary to pursue the inquiry any farther.

These then, are the principal varieties into which the problem of expansion divides itself, when considered merely as a property of elastic fluids; and when the several questions which we have discussed are properly understood, there can be no difficulty in applying them to the determination of the power, and regulating the parts and proportions of a steam-engine.

But although the topics hitherto considered, are those chiefly that require calculation in practice, in as far at least, as the principle of expansion only is concerned; yet there are various other circumstances to be taken into account, which have not heretofore received any share of our attention, and it is to these that we now proceed to address ourselves.

SECTION IV.

SINGLE-ACTING ENGINE WORKING BY EXPANSION AND CONDENSATION.

IN what we have hitherto done, we have taken no notice of the steam as pressing upon an air-tight piston, moving up and down in a close cylinder, so as to produce mechanical action; our inquiries have only extended to the examination of the several conditions that present themselves, when a certain quantity of steam is admitted into the cylinder under a given pressure, and then allowed to expand itself, so as to occupy any proposed number of times its original space, the vessel enlarging its capacity as the expansion proceeds, until an equilibrium obtains, and the temperature of the steam remaining constantly the same from the time it is admitted into the vessel until the dilatation terminates.

We must now however, extend our inquiries to the case, in which the steam, by alternately raising and depressing an air-tight piston fitted in a close cylinder, operates to produce mechanical movement, and this brings us to the contemplation of the effects produced by steam in passing through an engine under a given degree of expansion.

There are numerous sorts of steam-engines, but it is not the object of the present performance to consider them in all their generality, the plan is limited to the examination of those only which work by expansion; and in order to avail ourselves to the fullest extent of the advantages which this mode of working steam affords, we shall attentively examine the different kinds of engines in which it is employed, and in order to render the subject as clear and intelligible as possible, it becomes necessary to detail the various peculiarities to a considerable extent, and with some degree of care and attention. Engines that work by expansion are of two kinds; namely, condensing and non-condensing engines, and these again admit of several varieties, according to the manner of construction and the arrangement of their parts. Those however, which we mean to examine more particularly in the succeeding pages, are,

1. The single-acting expansion engine, chiefly used for raising water.
2. The double-acting expansion engine, condensing and non-condensing.
3. The combined cylinder expansive engine, according to Woolf's and Sims's arrangements.

The different kinds of engines here enumerated, will therefore be discussed in the order of their arrangement, and since the single-acting engine is the simplest in its operation, it is to this kind of engine that our investigations will in the first place be directed.

The usual construction and arrangement, as well as the operation of the single-acting engine being so generally understood by the mechanical class of engineers, and those individuals who are engaged in their management, we need not here enter into any detailed description of these particulars, our present object being to consider the effect as produced by working steam under a given degree of expansion.

When the single-stroke engine acts by expansion, the formula for ascertaining the mean pressure which is equivalent to the entire action of the steam, is essentially the same as that which we have hitherto employed for the purpose; but in extending the principles of expansion to the working of a steam engine, there are various points to be considered which had no place in our former discussions.

In order, therefore, to obtain the mean effective pressure of the steam, the pressure under which it is generated in the boiler must be diminished by an estimated amount of the resistance, friction of the parts of the engine, loss of force by cooling, leakage, and other causes; but as these corrections will be different under different circumstances, it would be difficult to assign a constant value for general application, it will therefore be sufficient for our purpose to assume a medium value for the sum of the corrections, and then, if the result be modified for the effect of imperfect condensation, the mean effective pressure will thus be obtained, from which the power of the engine may be had in the usual way. If the sum of the corrections be made equal to about four tenths of the force of steam in the boiler, the result will probably not be very far from the truth; the various items which go to the composition of this amount, it is not necessary here to particularize.

The rule for calculating the power of the single-acting engine is very simple, and is as follows.

> RULE.—*Multiply the mean effective power of the steam per square inch by the square of the diameter of the cylinder drawn into the constant decimal* 7854, *and by half the velocity of the piston in feet per minute, and the product will be the effective power of the engine estimated in pounds raised to the height of one foot; then, if this amount be divided by* 33000, *the result will be the number of horses by which the power of the engine is expressed.*

Example.—Let the force of the steam in the boiler be 48 lbs. to the square inch; of what power is the engine, supposing the diameter of the cylinder to be 60 inches, the length of stroke 7 feet, the velocity of the piston 270 feet per minute, and the steam to be cut off at one-fourth of the stroke, the pressure of the uncondensed steam being equivalent to $2\frac{1}{2}$ lbs?

Taking the loss of force by friction, cooling, leakage, &c., at four-tenths of the whole force under which the steam is generated in the boiler, we shall have the force with which it enters the cylinder $48 - 48 \times .4 = 28.8$ lbs. per square inch; this gives 17.18 lbs. for the mean pressure of the steam upon a square inch of the piston and this diminished by $2\frac{1}{2}$ lbs, for imperfect condensation, gives 14.68 lbs. for the mean effective pressure of the steam upon each square inch of the piston.

The area of the piston is 2827.44 square inches; for by the rules of mensuration we have $60^2 \times .7854 = 3600 \times .7854 = 2827.44$; and half the velocity of the piston in feet per minute is $\frac{270}{2} = 135$; consequently by multiplication we get $2827.44 \times 135 \times 14.68 = 5603420.592$ lbs., and this being divided by 33000, gives 169.8 for the number of horses equivalent to the power of the engine.

SECTION V.

THE DOUBLE-ACTING ENGINE, WORKING BY EXPANSION AND CONDENSATION.

HAVING thus shown by a numerical example, how to calculate the power of the single-acting engine, we need not consider it any farther in this place, the great simplicity of its operation rendering fuller discussion unnecessary; the next object that claims our attention, is the double-acting engine, or that in which the steam is alternately applied to act on each side of the piston, while a vacuum is formed on the other side.

This engine in its general construction, resembles the single one before described; it however differs from it in having a passage from the boiler, both to the top and the bottom of the cylinder, and a similar passage from both to the condenser; by this form of construction, no counterweight is required to raise the piston, nor does the steam pass from the upper to the lower side; the piston is impelled by the force of the steam in both directions, and for this reason, when an engine of this construction is compared with a single one of equal size, it requires a double quantity of steam, and developes a double power in the same space of time.

In order therefore to form a correct notion of the double acting engine when working expansively, we must conceive the steam to be generated in the boiler under a very high degree of pressure, and from thence to pass into the cylinder at another pressure considerably reduced below that under which it is generated, but still of sufficient intensity to overcome the accumulated resistances, and communicate motion to the engine, with the whole train of machinery to which it gives motion and mechanical effect.

The steam being admitted into the cylinder under the pressure just alluded to, it puts the piston into motion, and when it has traversed a certain portion of its range, the flow of steam from the boiler is intercepted, and that which is now in the cylinder expands itself, and by means of its elastic or expansive power, it carries the piston to the extremity of its stroke or range.

The operation is then repeated on the opposite side of the piston, and the expanded steam being allowed to flow into a separate vessel, in which a vacuum more or less perfect has been formed, it is there condensed by being subjected to the influence of cold water variously applied, thereby renewing the vacuum and fitting the engine for a repetition of its action, which is thus rendered continuous.

Thus the steam from the boiler being admitted into the cylinder alternately above and below the piston, will cause it to descend and ascend accordingly, the condensation being always effected on that side of the piston opposite to the action of the steam. These particulars, however, are so familiar to practical engineers, that it is needless to insist more at large upon them; we shall therefore proceed to show the method of computing the power of the double-acting engine, and point out the advantages to be derived from working steam expansively. The first object therefore, that claims our attention in the first part of the enquiry, is the method of computing the power of the double-acting engine when working by expansion, and for this purpose it becomes necessary to employ such symbols as will represent, generally, the several quantities that enter into the computation. Thus,

Let d = the diameter of the cylinder in inches,
v = the velocity of piston in feet per minute,
h = the number of horses equivalent to the power of the engine,
p = the mean pressure of the steam in pounds per square inch.

Then, by taking the number 33000 in pounds raised one foot per minute, as the representative of the horse's power, the equation which represents the power of the engine in general terms, becomes as follows, viz :—

$$h = 0.7854 d^2 p v \div 33000 \qquad \text{(G)}$$

And the same equation may be expressed in a specific form as follows:

$$\textit{horse's power} = 0.7854 \times \textit{diameter}^2 \times \textit{mean pressure} \times \textit{velocity} \div 33000. \qquad \text{(G')}$$

This is the form which the equation assumes, when the steam is admitted from the boiler throughout the whole length of the stroke, and the practical rule which it supplies may be expressed in words at length, as follows:

> RULE.—*Multiply the area of the piston in square inches, by the pressure of the steam in pounds per square inch, and again by the velocity of the piston in feet per minute; then divide the product by the constant number* 33000 *for the power of the engine.*

This rule is alike applicable to every kind of steam engine when the mean effective pressure of the steam is known; but, since that requires to be computed in the case of an engine working by expansion, it becomes necessary so to modify the above equation, as to involve the process for finding the mean pressure which is due to the entire effect of the steam; this process we have represented in equation (B), and when that value of p is substituted in the general equation above, we get,

$$h = 0.7854 d^2 P v \, (\text{hyp. log. } n + 1) \div 33000 \, n. \qquad \text{(H)}$$

This expression exhibits the whole of the steps that must be performed to determine the power of the engine, on the supposition that the values of all the letters which it involves are known, with the exception of the letter h, which indicates the number of horses that are equivalent to the power of the engine.

The form which the equation here assumes is a little complicated, and may perhaps offer some difficulty to practical men in general; it is nevertheless sufficiently simple as an instrument of calculation, and since we have elsewhere shown how to obtain the mean pressure due to the whole effect of the steam, the process will be greatly facilitated by the following practical rule.

> RULE.—*Calculate the mean pressure of the steam in pounds per square inch of the piston, by the rule to equation* (B) *or* (C); *then multiply altogether the mean pressure of the steam thus found, the area of the piston in square inches, and the velocity of the piston in feet per minute; then divide the product thus obtained by the constant number* 33000, *for the horses power of the engine sought.*

Now the rule as here expressed, can offer no difficulty whatever in its application, provided that the first part of it, which determines the mean pressure of the steam be well understood, for it is in that only that anything difficult can be apprehended, the rest of the computation requiring nothing beyond the common rules of arithmetic, rules which, it is presumed, are perfectly familiar to every person connected with the construction or management of a steam-engine.

It must be confessed, however, that the calculation for any particular case, being performed as indicated by the above formula and the rule deduced from it, is tedious and prolix, and since the process is greatly facilitated by the use of logarithms, it will be advantageous to many of our readers to express the rule in terms of these numbers also; but in the first place, it becomes expedient to adapt the formula to the use and application of logarithms; and accordingly, when so adapted, it is as below.

$$\ldots \log. \textit{horse's power} = \log. 0.7854 + 2 \log. \textit{diam.} + \log. \textit{mean pressure} + \textit{vel.} - \log. 33000. \quad \ldots \quad \text{(I)}$$

Here, as in the case above, the value of the mean pressure due to the entire effect of the steam must be determined by an independent operation, according to the rule deduced from equation (B) or (C), and when the mean pressure has thus been ascertained, the power of the engine will be found by the following rule.

> RULE.—*Find the mean pressure of the steam as directed in the rule to equation* (B) *or* (C); *then add together, the logarithm of the mean pressure; twice the logarithm of the diameter in inches; the logarithm of the velocity of the piston in feet per minute; the logarithm of the decimal number* 0.7854, *and the arithmetical complement of the logarithm of* 33000; *then the natural number answering to the sum of all these, will be the horses power of the engine.*

We shall now work out an example by each of these rules, as a comparison of the operations will serve to show in what respects the one is to be preferred to the other in point of facility and ready application.

Example.—The cylinder of a double-acting condensing engine working expansively is 30 inches diameter, the length of the cylinder or stroke 7 feet, and the number of double strokes per minute 20; what is the effective power of the engine, supposing the steam to enter the cylinder under a pressure of 35 pounds to the square inch, and to act at full pressure for two-sevenths of the length of the stroke?

By the rule to equation (H), we have first to calculate the mean pressure of the steam on a square inch of the piston; the initial pressure, or that under which it enters the cylinder, being equivalent to 35 pounds per square inch. Now the rule to be applied for this purpose is that given under equation (C), and by it we get $n=\frac{7}{2}=3.5$, and $P'=\frac{35\times2}{7}=10$; that is, the index of expansion is 3.5, and the terminal pressure of the steam is 10 pounds per square inch of the piston. But the hyperbolic logarithm of 3.5 is, 1.252763, which being increased by unity, gives $1.252763+1=2.252763$, and this multiplied by 10 gives 22.52763 pounds for the mean pressure due to the entire effect of the steam.

This, however, is on the supposition that the condensation of the steam is perfect; a condition which never takes place even in the best-constructed engines; indeed the temperature of the uncondensed steam is seldom less than 120 degrees, and that gives a pressure of about 3.7 lbs. per square inch acting on the other side of the piston, which must be deducted from the mean pressure as obtained above, to get the effective mean pressure for calculating the precise power of the engine; hence we have $22.52763-3.7=19.82763$ pounds per square inch.

Having thus determined the mean pressure of the steam, the remaining part of the operation, according to the first of the foregoing rules, will be as follows.

Area of the piston$=30\times30\times0.7854=706.86$ square inches.
Velocity of the piston$=7\times2\times20=280$ feet per minute.

Then we get $19.82763\times706.86\times280=3924300.24$, and if this be divided by 33000, we get $3924300.24\div33000=118.92$ horse power.

That part of the calculation which determines the mean pressure of the steam must be precisely the same by both rules; the process therefore need not be repeated. Hence by the second of the foregoing rules we have,

Mean pressure, 19.82763 . .		log. 1.2972708
Diameter of the piston, 30 .	2	log. 2.9542708
Velocity of the piston, 280 . .		log. 2.4471580
Constant decimal, 0.7854 . .		log. 9.8950899
Constant for horse power, 33000	or co.	log. 5.4814861
Therefore the horses power of the engine is 118.92, the same as before .		log. 2.0752756

In this example, the pressure of the steam as it enters the cylinder is 35 pounds per square inch, and this gives a mean pressure, on the supposition of complete exhaustion, of 22.52763 pounds. But the arithmetical mean between the initial and terminal pressure is 22.5; it is therefore obvious that there is a point at which the full pressure steam may be intercepted, so that these means may be equal to one another, and since the fraction which indicates this point must be the same for every length of cylinder, it may be both useful and interesting to show, in what manner the proper point of interception is to be ascertained. In order however, to prove the accuracy of our reasoning on this subject, it becomes necessary in the first place, to compute the mean pressure for two different points of interception, between which the required point is situated.

For this purpose, let the two assumed points be at one-sixth and two-sixths the length of the cylinder respectively, and suppose the initial pressure of the steam on entering the cylinder, to be equivalent in each case to 36 pounds upon a square inch of the piston; then it is obvious, that when the flow of steam from the boiler is intercepted at one-sixth of the stroke, it must by expansion occupy six times the original space, and exert a pressure of one-sixth of its original force, amounting to 6 pounds on the square inch. But on the other hand, when the full pressure steam is intercepted at two sixths of the stroke, it will by expansion occupy only three times the original space, which gives a terminal pressure of 12 pounds to the square inch.

Now, in the case where the steam is intercepted at one-sixth of the stroke, the mean pressure due to the entire effect of the steam, as computed by the rule to equation (B), or (C), is as below.

$$\textit{mean pressure} = \frac{36}{6}(\text{hyp. log. } 6+1) = 6 \times 2.7917595 = 16.751 \text{ lbs.}$$

But the arithmetical mean between the initial pressure of 36 pounds to the square inch, and the terminal pressure of 6 pounds, is $\frac{1}{2}(36+6)=21$ pounds per square inch, being in excess of the mean as computed above, by about $4\frac{1}{4}$ lbs. per square inch.

Again, in the case where the steam is intercepted at two-sixths of the stroke, the mean pressure due to the entire effect of the steam, is as under.

$$\textit{mean pressure} = \frac{36}{3}(\text{hyp. log.} + 1) = 12 \times 2.0916123 = 25.1833476 \text{ lbs.}$$

In this case, the arithmetical mean between the initial and terminal pressures, is $\frac{1}{2}(36+12)=24$ pounds per square inch, being in defect of the mean as computed above, by 1.1833476 lbs. per square inch.

It therefore appears, that when the steam is intercepted at one-sixth of the stroke, the arithmetical mean between the initial and terminal pressures, is greater than the computed mean; but when the steam is intercepted at two-sixths of the stroke, the computed mean exceeds the arithmetical mean; the point at which these means are equal, must therefore be situated between one-sixth and two-sixths the length of the stroke, and it is now our object to determine where that point obtains.

According to the notation which we have hitherto employed, the arithmetical mean between the initial and terminal pressures, is

$$\tfrac{1}{2}(\mathrm{P}+\mathrm{P}') = \frac{\mathrm{P}(n+1)}{2n},$$

and the mean which is equivalent to the entire effect of the steam, as represented by equation (B), is

$$p = \frac{\mathrm{P}}{n}(\text{hyp. log, } n+1)$$

But by the conditions of the question, these are to be equal; therefore by comparison, we get

$$\frac{\mathrm{P}(n+1)}{2n} = \frac{\mathrm{P}}{n}(\text{hyp. log. } n+1)$$

Now, the term $\frac{\mathrm{P}}{n}$, or the terminal pressure of the steam, being common to both sides of the equation, it may be left out of each, and the equation will still obtain; thus we have $\frac{1}{2}(n+1) = \text{hyp. log. } n+1$; and finally by reduction it is

$$n - 2 \text{ hyp. log. } n = 1. \qquad \text{(K)}$$

This very simple and elegant expression points out an important fact in the theory of the steam engine, such as perhaps would not occur to practical engineers without the aid of analysis, but which, if universally recognised as a maxim in the construction of the expansive engine, might tend greatly to simplify the calculation of its power.

The fact to which we allude, consists in the circumstance of the expression that we have just derived, being altogether independent of the pressure of the steam, either in the initial or the terminal state; from which we infer that the value of *n*, (or the denominator of the fraction, which indicates at what point of the stroke the steam is intercepted,) is constant for every length of cylinder; but in order to simplify the determination of its value, it will be convenient to give the formula in terms of the common logarithms. Thus we have

$$n - 2 \times 2.3025851 \text{ log. } n = 1; \text{ that is, } 0.4342945\, n = 0.4342945 \quad . \quad \text{(L)}$$

Now, in order to reduce this equation, we have only to find a number or quantity such, that, being multiplied by the fraction 0.4342945, and the product diminished by twice the logarithm of the said number or quantity, the remainder shall be equal to the fraction 0.4342945.

It may be proper here to state, that the constant quantity 0.4342945, is the modulus of the common system of logarithms; or it is the logarithm of the number 2.7182818, of which the hyperbolic logarithm is 1; or it is the reciprocal of 2.3025851, which is the hyperbolic logarithm of 10.

This being premised, we may furthermore remark, that since the required quantity is expressed by two terms of different affections, or such as are not comparable, its value can only be determined by approximation, or by trial and error; but since the form of the expression which involves it is so exceedingly simple, no difficulty can arise in its reduction if the common operations of arithmetic be properly understood.

We have already determined the limits between which the value of the required quantity lies, and it only now remains for us to assign the nearest approximate value in the simplest terms possible; and for this purpose, we have only to assume a series of consecutive terms between the computed limits, those terms differing from each other by a very small interval; then if each term of the assumed series be substituted for the symbol *n* in the above expression which involves its value, some one or other of the results will be found to approximate to the fulfilment of the required condition; viz. that of 0.4342945 times the required quantity diminished by twice its logarithm, shall be equal to 0.4342945.

Let us therefore begin with the nearest limit which is 3, as we know by the error which the assumption of that number produces; then let us take a series of terms differing from each other by the common interval of one-tenth of the unit, so that the series of terms is 3.0; 3.1; 3.2; 3.3; 3.4; 3.5; 3.6, &c.; as far as it is necessary to extend the terms; hence we have as follows.

1. $3.0 \times 0.4342945 - 2 \times 0.4771213 = 1.3028835 - 0.9542426 = 0.3486409$
2. $3.1 \times 0.4342945 - 2 \times 0.4913617 = 1.3463130 - 0.9827234 = 0.3635896$
3. $3.2 \times 0.4342945 - 2 \times 0.5051500 = 1.3897424 - 1.0103000 = 0.3794424$
4. $3.3 \times 0.4342945 - 2 \times 0.5185139 = 1.4331719 - 1.0370278 = 0.3961441$
5. $3.4 \times 0.4342945 - 2 \times 0.5314789 = 1.4766013 - 1.0629578 = 0.4136435$
6. $3.5 \times 0.4342945 - 2 \times 0.5440680 = 1.5200308 - 1.0881360 = 0.4318948$
7. $3.6 \times 0.4342945 - 2 \times 0.5563025 = 1.5634602 - 1.1126050 = 0.4508552$

Now, by comparing these results respectively, with the absolute quantity 0.4342945 as given in the equation, we find that it exceeds all the terms below 3.6, but is exceeded by the result corresponding to that term; it therefore appears, without extending the series to the greater limit, that the value of n lies between 3.5 and 3.6; consequently, by substituting each of these quantities or value of n instead of it in the foregoing equation (L), we shall get the following results, viz.

When $n=3.5$; then, $0.4342945\,n - 2 \log. n = 0.4318948$.
When $n=3.6$; then, $0.4342945\,n - 2 \log. n = 0.4508552$.

Here it is manifest, that by the one assumption, the result is too little by 0.0023997; for $0.4342945 - 0.4318945 = 0.0023997$; but according to the other assumption, the result is too great by 0.0165607; for $0.4508552 - 0.4342945 = 0.0165607$; consequently by the rule of double position, we have as follows.

First assumption, . $3.5 \times .0165607 = 05796245$, product.
Second assumption, $3.6 \times .0023997 = 00863892$, product.

Adding these two products into one sum we get, 0.06660137; and in like manner, by taking the sum of the errors, we get 0.0189604; then, by considering both these sums as integers, we obtain by division,

$$n = 6660137 \div 1896040 = 3.513 \text{ very nearly.}$$

It therefore appears, that if steam be admitted into the cylinder at full pressure during the $\frac{1}{3.513}$th part of the stroke, the mean pressure due to the entire effect of the steam, will be precisely the same, as the arithmetical mean between the initial and terminal pressures; for in that case the steam will be dilated 3.513 times.

It may be remarked with regard to the method of solution here employed, that some other of the approximating rules might have been adopted, which would have brought out the result more expeditiously, and probably too with greater accuracy; but as the rule of position is given in most books of common arithmetic, and is of course accessible to every reader, we think it likely to be more generally understood, and it is for this reason only that we have employed it in the present instance, in preference to other methods of greater accuracy.

If the calculation had been performed by the first arrangement of the equation, or that marked (K), the operation would have been attended with less labour, but the result would nevertheless have been precisely the same; the exhibition of the process however, may be more satisfactory to the general reader, than the mere verbal statement of the fact, and for this reason, we think proper to repeat the calculation as underneath.

1. $3.0-2\times 1.0986123=3.0-2.1972246=0.8027754.$
2. $3.1-2\times 1.1314021=3.1-2.2628042=0.8371958.$
3. $3.2-2\times 1.1631508=3.2-2.3263016=0.8736984.$
4. $3.3-2\times 1.1939224=3.3-2.3878448=0.9121552,$
5. $3.4-2\times 1.2237754=3.4-2.4475508=0.9524492,$
6. $3.5-2\times 1.2527629=3.5-2.5055258=0.9944742.$
7. $3.6-2\times 1.2809338=3.6-2.5618676=1.0381324.$

Here as in the preceding case, we find that the value of n lies between the sixth and seventh terms, or between 3.5 and 3.6;—consequently, by substituting these numbers respectively for n in equation (K), we shall get the following results, viz.

When $n=3.5$; then, $n-2$ hyp. log. $n=0.9944742$.
When $n=3.6$; then, $n-2$ hyp. log. $n=1.0381324$.

Where it is obvious that the one is less and the other greater than unity, the errors being 0.0055258, and 0.0381324, the one being too little, and the other too great; consequently, by the rule of double position, we get

First assumption, . $3.5\times 381324=13346340$, product.
Second assumption, $3.6\times 55258=1989288$, product.

Therefore, dividing the sum of the products by the sum of the errors, we have

$$n=\frac{15335628}{436582}=3.513, \text{ the same as before.}$$

The least vulgar fraction that will represent the portion of the stroke traversed by the piston before the steam is cut off is $\frac{1250}{4391}$, being a little less than $\frac{2}{7}$, but in practice the decimal form had better be retained for the more ready application of the logarithms.

It now remains for us to show, that the value of n which we have obtained above, is that which satisfies the conditions of the problem; namely, that mean pressure due to the entire effect of the steam, is the same as the arithmetical mean between the initial and the terminal pressures.

In order to this, let us suppose that the initial pressure is 35.127 pounds upon a square inch of the piston; then, since the steam dilates itself 3.513 times according to the foregoing calculation, the terminal force or pressure will be 10 pounds upon the square inch.

Now, the hyperbolic logarithm of 3.5127, is 1.2563850, which being increased by unity and multiplied by 10 the terminal pressure, as directed by the rule to equation (C), gives

$$\textit{mean pressure} = 10(1+1.256385) = 22.56385 \text{ pounds.}$$

This is very nearly equal to half the sum, or the arithmetical mean of the initial pressure 35.127, and the terminal pressure of 10 pounds upon the square inch; for it is

$\frac{1}{2}(35.127+10)=22.564$ pounds, agreeing to the third place of decimals with the mean as calculated by the general rule.

The object of this inquiry, is to simplify the calculation of the power of the engine when working by expansion, a point which is gained, by determining the mean pressure due to the entire effect of the steam, independently of the use of hyperbolic logarithms; for in this way, we have only to take half the sum of the initial and terminal pressures, which will be the mean pressure very nearly; or if we multiply the initial pressure by the fraction 0.6423, the product will be the mean pressure very nearly; thus for example, $35.127 \times 0.6423 = 22.562$ pounds, differing very little from the foregoing result.

This rule for finding the mean pressure however, can only be regarded as nearly accurate, when the cut-off approaches to two-sevenths of the stroke, which we believe is nearly the proportion for using expansive steam to the greatest advantage. This however, constitutes the practical limit of the rule, and consequently, for any other degree of expansion that may happen to be employed, the mean pressure must be computed by the general rule; but admitting the expansion to be very nearly as here stated, the number of horses equivalent to the power of the engine, is elegantly indicated as follows.

$$h = 0.5044\, d^2 P\, v \div 33000 \qquad \text{(M)}$$

Where the several symbols d, h, P and v denote the quantities already specified in the notation; that is, d the diameter of the cylinder in inches; h the number of horses equivalent to the power of the engine; P the initial pressure of the steam, and v the velocity of the piston in feet per minute.

The foregoing equation, when exhibited in a specific form, would stand as under.

$$\text{The } \textit{horses' power} = 0.5044\, \textit{dia.}^2 \times \textit{initial pressure} \times \textit{vel. pist.} \div 33000 \quad \text{(N)}$$

And the practical rule which it supplies, may be expressed in words as follows.

> RULE.—*Multiply* 0.5044 *times the square of the diameter of the cylinder in inches, by the initial pressure of the steam in pounds per square inch, and again by the velocity of the piston in feet per minute, then divide the product by the constant number* 33000 *for the power of the engine sought.*

Taking therefore the data of the example under equation (I), the power of the engine will be determined as follows.

$$0.5044 \times 30 \times 30 = 453.96;$$
$$453.96 \times 35 \times 280 = 4448808;$$

consequently, by division, we obtain

The horses' power $= 4448808 \div 33000 = 134.81$.

The power of the engine however, when calculated by the rule for $n = \frac{2}{7}$, is only 118.92 horses, being a difference of 15.89 horses, for which the difference of $n = 3.5$ and $n = 3.5127$ very satisfactorily accounts, taking the initial pressure in both cases to be 35 pounds per square inch.

We have been the more particular in illustrating this case, for the purpose of showing the practical engineer, and those who have the management of steam engines, that when the cut-off is nearly two-sevenths of the stroke, the power of the engine may be very nearly approximated, without a separate calculation for the mean pressure of the steam, which is the chief difficulty that presents itself in the general computation.

Returning to the equation (H) which represents the power of the engine, we are enabled to determine therefrom, the diameter of the cylinder, so that the engine shall be of any proposed power, on the supposition that the initial pressure of the steam, the number of times it is dilated, and the velocity of the piston are given.

Now this is a very important case of the general problem, and ought to be well understood by every person connected with the construction and management of the steam engine; for it is a well-known fact amongst practical mechanics, that the power of an engine depends on three data only, and these are as follows.

1. *The diameter of the steam cylinder in inches.*
2. *The velocity of the piston in feet per minute.*
3. *The mean effective pressure of steam upon a square inch of the piston.*

From these data conjointly the power of the engine can readily be computed; and when any two of them, together with the power of the engine are given, the remaining one can always be found. It is therefore a matter of the utmost importance, that the engineer should be able to determine any one of these quantities when the others are known.

In expansive engines, the mean effective pressure of the steam is a subject of computation. It is obtained from the initial pressure, or that exerted by the steam before expansion begins, and the number of times that the steam dilates itself into the cylinder; or which amounts to the same thing, the part of the stroke which is performed under full pressure.

Since therefore, the mean effective pressure of the steam is determined by a distinct operation, it may be as well to consider it as being known *a' priori*, and to substitute its appropriate symbol in the general equation (H), for the power of the engine; this will carry us back to equation (G), in which all the factors are represented in simple terms only, the compound expressions for the mean effective pressure of the steam, being replaced by the symbol p, the character adopted for it in our original notation.

The formula here alluded to, and thus particularly brought into notice, is as under, viz.

$$h = 0.7854\, d^2 p v \div 33000.$$

Now this is precisely the form of equation, which is employed for computing the power of a double-acting condensing engine, when the steam is used at full pressure throughout the length of the stroke; so that, with the exception of having to calculate the mean effective pressure of the steam for the expansive engine, the rule for determining the power, is in both cases the same.

Supposing that in the above equation, the values of h, p, and v are known, and it is required to determine the corresponding value of d, so that the three proposed values may certainly obtain.

Here then it is obvious, that before the value of d can be expressed in known terms, it must be disengaged from all the quantities with which it is combined; that is, it must be made to occupy one side of the equation, when uncombined with any of the other quantities involved in the equation. Therefore, in order to disengage the quantity d, or the diameter of the cylinder, from the other factors where values are known, we must in the first place, express the momentum of the engine in pounds raised one foot per minute, and this is effected by multiplying the number of horses by 33000, the conventional unit for a horse's power.

Now, this number of pounds raised one foot per minute, is equal to the area of the piston in square inches, multiplied by its velocity in feet per minute, and again by the effective pressure of the steam in pounds per square inch. Consequently, if the number of pounds raised one foot per minute, or which is the same thing, if the entire product of the three quantities above-named, be divided by the product of any two of them, the quotient will manifestly be equal to the third or remaining quantity; hence it follows, that if the power or momentum of the engine in pounds, be divided by the product of the mean effective pressure power of the steam in pounds per square inch, and the velocity of the piston in feet per minute, the quotient will be the area of the piston in square inches.

But by the rules of mensuration, the area of the piston is expressed by the square of the diameter multiplied by the decimal 0.7854; hence, conversely, the square of the diameter, is equal to the area divided by the same decimal; and having the square of the diameter, the diameter itself is found by extracting the square root. This is the legitimate mode of analysing the general equation in reference to the diameter of the cylinder, and when the analysis is exhibited in a specific form, it is as underneath.

$$\textit{diameter} = 204.98\sqrt{\textit{horse power} \div \textit{mean pres.} \times \textit{vel. pist.}}$$

It may be proper to remark, that the co-efficient 204.98, is found from the division of 33000 by 0.7854, and afterwards extracting the square root; that is,

$$204.98 = \sqrt{\frac{33000}{0.7854}}$$

From the equation in the form which it now assumes, the following practical rule may be derived.

> Rule.—*Divide the number of horses power of the engine, by the effective mean pressure of the steam in pounds per square inch, multiplied by the velocity of the piston in feet per minute; then multiply the square root of the quotient by the constant number* 204.98, *obtained after the manner represented above, and the product will be the diameter of the cylinder in inches.*

Example.—In a double-acting expansive engine working by condensation, with the vacuum gauge at 29 inches; the steam is admitted into the cylinder at a pressure of 40 lbs. per square inch, and is intercepted at one-fourth of the stroke, the other three-fourths being effected by the expanding force of the steam; what is the diameter of the cylinder, supposing the power of the engine to be equal to 135 horses, and the velocity of the piston 280 feet per minute?

In this example, the steam exerts a pressure of 40 pounds per square inch during one-fourth of the stroke; it then begins to expand, and by diffusing itself into four times the space, terminates the stroke with a pressure of only 10 pounds to the square inch. Consequently, the mean pressure as calculated by the rule to equation (B) or (C), is

$$\textit{mean press.} = 10\,(\text{hyp. log. } 4 + 1) = 10 \times 2.3862944 = 23.863 \text{ lbs.}$$

This is the mean pressure exerted by the steam during the operation, on the supposition of a perfect vacuum; but by the question, the vacuum is only 26 inches of the mercurial gauge, and consequently the resistance of the uncondensed steam, is equal to four inches of mercury, or half a pound upon the square inch; the mean effective pressure of the steam is therefore only equal to $23.863 - .5 = 23.363$ pounds. Now the power of the engine as given by the question, is 135 horses, or it is $135 \times 33000 = 4455000$ pounds raised one foot per minute; therefore we have, according to what we have said above, the following comparison;

$$d^2 \times 23.363 \times 280 \times 0.7954 = 4455000;$$

from which the square of the diameter can readily be found, or by working as directed in the above rule, we get

$$\textit{diameter} = 204.98\sqrt{135 \div (23.363 \times 280)} = 29.45 \text{ inches.}$$

It is obvious from the nature of the numbers involved in this expression, that the actual operation by common arithmetic would be very operose, we shall therefore show the process as performed by logarithms,

Power of the engine, 135 horses . . .	log.	2.13 03338	add,
Under pressure of the steam, 23.363,	ar. co. log.	8.6314900	
Velocity of the piston, 280 . . .	ar. co. log.	7.5528420	
	2	18.3146658	=sum
		9.1573329	=half sum.
Constant co-efficient 204.98	log.	2.3117120,	add
Natural number, 29.45 inches	log.	1.4690449.	

Now this operation is as simple as can be wished for, and when we consider the importance of the element obtained by it, the great utility of the rule to practical engineers will at once be admitted.

From what has been done above in regard to the diameter of the cylinder, it is obvious, that by a similar transposition of the general equation (H), the value of any other of the quantities involved in it, can be determined in terms of the rest; but the transposition in the other case will be more easily effected, inasmuch as the resulting expression will not be encumbered with a radical or surd member.

We have already shown how to calculate the power of the engine by working in two different ways the example under the general equations (H) and (I) representing it; but the equation involves two other factors which may sometimes require a separate determination. These are,

1. *The mean pressure due to the entire effect of the steam.*
2. *The velocity of the piston in feet per minute.*

In order therefore to determine each of these quantities separately, it is necessary that the values of all the other quantities which enter the equation should be previously known, so that by disengaging the required member after the manner indicated by the combination, its value may also become known in terms of the rest.

Therefore to disengage the mean effective force of the steam, we must divide the momentum or mechanical effect of the engine in pounds raised one foot per minute, by the area of the piston in square inches, drawn into velocity of its movement in feet per minute, and the quotient arising from the division will be the mean pressure sought.

Now, the momentum of the engine expressed in pounds raised one foot per minute, is equal to the number of horses multiplied by 33000; and the area of the piston is expressed by the square of the diameter in inches, multiplied by the number 0.7854; consequently, we get

$$p=\frac{33000\,h}{0.7854\,d\ v}=\frac{42016.9\,h}{d^2 v}$$

This is the expression for the mean force developed by the steam in passing through the engine, and when reduced to a specific form, it is as underneath.

mean pressure=42016.9 × *horse's power* ÷ *diam.*^{2} × *vel. pist.*

The practical rule which this equation supplies, may be expressed in words at length in the following manner :—

> RULE.—*Multiply the horses power of the engine by the constant number* 42016.9, *and divide the product by the square of the diameter of the cylinder in inches, multiplied by the velocity of the piston in feet per minute, and the quotient will be the mean pressure of the steam in pounds per square inch.*

Example.—The diameter of the cylinder is 30.44 inches; the velocity of the piston 280 feet per minute, and the power of the engine equivalent to 135 horses; what is the mean pressure of the steam upon a square inch of the piston?

Here by the rule we have

$$h \times 42016.9 = 135 \times 42016.9 = 5672281.4, \text{ dividend};$$

$$d^2 = 30.44 \times 30.44 = 926.5936;$$

$$d^2 v = 926.5936 \times 280 = 259446.6, \text{ divisor}.$$

Therefore finally, if the first of these products be divided by the last, we have

$$\textit{mean pressure} = \frac{5672281.4}{259446.6} = 21.863 \text{ pounds per square inch.}$$

This pressure is also expressed by the term $\frac{P}{n}$ (hyp. log. $n+1$) + the effect of the uncondensed steam, which in the present instance is equivalent to 2 pounds upon the square inch ; hence it is,

$$\frac{P}{n}(\text{hyp. log. } n+1) = 23.863.$$

Which condition is satisfied when $P=40$ and $n=4$, as we have already seen by a previous computation.

Next for the velocity of the piston in feet per minute, we must again have recourse to the general equation, and by disengaging the term v by division as in the previous case, we get

$$v = \frac{33000 \times h}{0.7854 d^2 p} = \frac{42016.9 \times h}{d^2 p}$$

This equation for the velocity, is equally as simple as that for the mean pressure, and when expressed in specific terms, it is

$$\textit{velocity of piston} = 42016.9 \textit{ horses power} \div \textit{diam.}^2 \times \textit{mean press.}$$

> RULE.—*Multiply the horses power of the engine by the constant numnumber* 42016.9, *and divide the product by the square of the diameter in inches, multiplied by the mean pressure of the steam in pounds per square inch, and the quotient will be the velocity of the piston in feet per minute.*

Let the power of the engine be 135 horses, the diameter of the cylinders 30.44 inches, and the mean pressure of the steam 21.863 pounds per square inch as computed above; what is the corresponding velocity of the piston in feet per minute?

Here by the rule we have,

$$h \times 42016.9 = 135 \times 42016.9 = 5672281.4, \text{ dividend};$$

$$d = 30.44 \times 30.44 = 926.5936;$$

$$d^2p = 926.5936 \times 21.863 = 20258.116, \text{ divisor}.$$

Therefore, if the dividend be divided by the divisor, the quotient will give the velocity as follows.

$$vel.\ pist. = \frac{5672281.4}{20258.116} = 280 \text{ feet per minute.}$$

When all that we have done has been properly studied and well understood, it will readily be admitted, that we have developed the doctrine of expansion to a sufficient extent for every practical purpose, in as far at least as the single cylinder engine is concerned, while it is supposed to work by expansion and condensation; but there are other circumstances to be taken into account before our theory can be pronounced complete, and it is to these that our attention must now be directed.

In the first place then, since we possess the power of regulating the generation of steam in the boiler, in such a manner, as to admit it into the cylinder under any proposed pressure; it follows, that we can so adjust the point of cut-off, as to give to the terminal pressure any value at pleasure.

This is indeed a very important circumstance as regards the proper management of the steam engine when working expansively; for if under certain conditions, the terminal pressure of the steam should be allowed to descend below a given point, it might so happen that the piston would not reach the extremity of its range, even admitting the assistance derived from the power concentrated in the fly-wheel; and consequently, a stoppage of the engine and the machinery would be the result; at any rate, a shock would be given of such a nature and force, as would be very apt to cause a failure of some particular part.

In order therefore, to guard against such a contingency, the terminal pressure of the steam should never be less than that which would be sufficient to work the engine when admitted at full pressure throughout the stroke; for if otherwise, much would be left to be effected by the momentum of the machinery itself.

Thus for instance, if a common double-acting engine, be found just capable of performing its work with a constant pressure of 7 pounds to the square inch of the piston, it would not be prudent to permit the terminal pressure of a similar engine working expansively, to descend much below that point, otherwise the engine might be apt to stop, as soon as the dilatation has reached that state, in which the pressure is just sufficient to keep it in motion when acting constantly with the same intensity, even although the mean pressure should be very considerably greater.

Now, we have elsewhere stated, that the initial pressure of the steam as it enters the cylinder, when divided by the index of expansion, is equal to the terminal pressure; hence it follows conversely, that the initial pressure divided by the terminal pressure, will give for a quotient the index of expansion, or the number of times that the steam is dilated in the cylinder; and if this number be made the denominator of a fraction of which the numerator is unity, it will represent the part of the stroke performed at the instant the steam is intercepted.

To illustrate this, suppose the steam as it is received from the boiler, to enter the cylinder under a pressure of 35 pounds per square inch; at what part of the stroke ought it to be intercepted, in order that the terminal pressure when fully expanded, may be 7 pounds per square inch?

Here it is obvious from what has been said above, that if the initial pressure of 35 pounds, be divided by the terminal pressure of 7 pounds, the quotient will be the index of expansion required; therefore we get

$$n=\frac{P}{P'}=\frac{35}{7}=5, \text{ the exponent sought.}$$

Consequently, one-fifth of the stroke must be traversed by the piston under the full pressure of the steam, before the flow from the boiler is intercepted;

so that if the length of the stroke be 10 feet, the steam must be cut off when the piston has passed over two feet, the other eight feet of the stroke being effected by the expansive force of the steam under a mean pressure of 11.266 lbs. per square inch, the mean due to the total effect of the steam being equal to 18.266 lbs., as calculated by the rule to equation (B) or (C).

When any other terminal pressure is given than that which we have here illustrated, it is obvious that the point of cut-off will be found in a similar manner when the initial pressure is given; but another question may arise in reference to the mean pressure, and this as regards the expansive engine working by condensation, is probably the most important case of the two, since this mean pressure is one of the elements on which the power of the engine depends, irrespective of the point of cut-off, and must therefore always be known *a' priori,* in order to determine the actual power of the engine.

Taking as in the previous case, the initial pressure equal to 35 pounds per square inch; at what point in the length of the stroke, ought the steam to be intercepted, in order that the mean pressure may be 18.266 pounds upon a square inch of the piston.

The solution of this case of the question manifestly depends upon the reduction of the general equation (B), which being reduced to a form resolvable by common logarithms, it gives us

$$0.4342945 \times n \times p = \text{P} \ (\log.\ n + 0.4342945).$$

There is a form in which the equation has already been represented in the foregoing pages, and when the numerical values of P and p are respectively substituted instead of them, the expression in its lowest terms becomes

$$n - 4.4121 \log.\ n = 1.9161.$$

Therefore in reducing this equation in reference to n, it is only necessary to find a number such, that being diminished by 4.4121 times its common logarithm, the remainder shall be 1.9161.

Now, in the present instance the required number is 5, for the common logarithm of 5 is 0.69897, and this being multiplied by 4.4121, gives 3.0839; therefore by subtraction we get

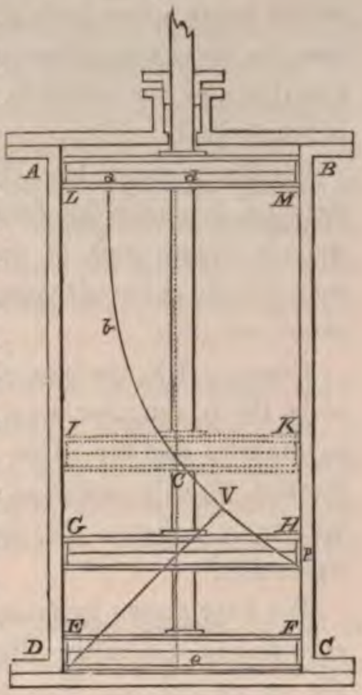
$$n = 5 - 3.0839 = 1.9161.$$

The importance of these calculations however, will be best appreciated when referred to a diagram illustrative of, and connected with, some particular case.

Let ABCD *fig.* 10, be the cylinder of a steam engine, into which the steam is admitted from the boiler under a given pressure, and in which a steam-tight piston ascends and descends alternately to the extent of 10 feet; that is making a ten feet stroke both upwards and downwards.

Let us in the first instance suppose the piston to be at the bottom of the cylinder, or in the position represented by EF; then, when steam from the boiler is admitted underneath it, it will ascend to another position represented by GH, and will continue to ascend with the same force as long as the steam is permitted to flow; but when the steam is intercepted, the force of ascent will be diminished, and the piston is carried to the top of the cylinder at LM by the direct power of expansion alone, the effect thus produced being wholly additional to that derived from the steam under its initial intensity.

Fig. 10.



In the example to which the above diagram refers, the steam from the boiler is intercepted when the piston has traversed 2 feet of the length of stroke, for by the calculation, we found the value of n, or the index of expansion to be equal to 5 exactly, and the whole length of the cylinder being divided by 5, gives 2 feet for the distance traversed by the piston before the flow of steam from the boiler is intercepted. Consequently, when the piston has ascended to the position GH, or 2 feet from the bottom of the cylinder, the steam is pressing on its under surface with a force of 35 pounds to the square inch; but at that point the supply of steam from the boiler is instantaneously cut off, and from thence it raises the piston by endeavouring to expand itself into a greater space, the elastic force decreasing in proportion as the space which it occupies increases; and when the piston has attained the top of its range, the steam has expanded itself into five times the space which it occupied under full pressure, and the terminal pressure is reduced to seven pounds on the square inch.

The pressure of the steam has therefore decreased from 35 pounds upon the square inch, its original intensity, to seven pounds, having diminished in consequence of the expansion to the extent of 28 pounds per square inch, while the piston traversed eight feet of the stroke, a degree of variation that would imply a very variable and irregular motion; this however, is not the case, for the change of motion even in very long strokes, is scarcely perceptible, and it is now our object to show in what manner this uniformity of motion is maintained.

For this purpose, let GH the diameter of the cylinder in the foregoing drawing, be made the linear representative of the pressure of the steam upon a square inch of the piston in that particular position; then P a point in the curve of expansion is known, inasmuch as the ordinate at that point is known.

Conceive EF, the base of the cylinder produced, and the vertical side EL to be the rectangular asymptotes of the common hyperbola, and with these asymptotes and the given point P, let the hyperbola be delineated by the method already explained for that purpose, and the curve thus obtained, will truly indicate the character of the variable force developed by the expansion.

We have shown by means of a previous computation, that the mean pressure due to the entire effect of the steam, is equal to 18.266 pounds upon the square inch, and from this, combined with the property of the hyperbola, we can readily find the position of the piston, when the steam is exerting that degree of force.

Thus we have $\frac{35\times 2}{18.266}=3.832$ feet, so that when the piston is in the position IK, the steam operates on it with a pressure of 18.266 pounds per square inch, and this is the element by which, when considered to act uniformly, the power of the engine is to be computed in the manner already exemplified.

From the same scale of equal parts, on which the diameter GH represents 35 pounds, and on the diameter IK, set off the computed number 18.266, and through the point c in which the measured distance intersects the diameter, draw the vertical line ecd to meet the diameter LM in the point d; then is dM equal to $35-18.266=16.734$ pounds, the force lost by the steam while the piston ascends from the position GH to IK, or through the space $\text{GI}=1.832$ feet.

It therefore appears, that immediately after the expansion begins, the elastic force of the steam decreases very rapidly, but when the piston has passed the point where the mean pressure obtains, the rate of decrease is much slower, the difference in the variations becoming smaller and smaller, and the pressures approximating much nearer to a state of uniformity.

All this is manifest from an inspection of the curve, for as we have already seen, the portion of the diameter d M, is the linear representative of 16.734 pounds, and that is the decrease of pressure occasioned by the steam expanding through the space HGIK, or while the piston traverses the space GI equal to 1.832 feet; but while the piston traverses the remaining portion of its range IL equal to 6.168 feet, the pressure is decreased 11.266 pounds, for $18.266-7=11.266$; from which we infer, that the farther the expansion is carried, the more uniform does the pressure of the steam become, and the more slowly does it approach to its limit.

Thus for instance, if the length of the cylinder were 20 feet instead of 10, as we have supposed it; then the steam being cut off at the same point, or when the piston has traversed 2 feet of the length, it would expand itself ten times in diffusing itself throughout the cylinder, and the terminal pressure would be $3\frac{1}{2}$ pounds per square inch, instead of 7 pounds as in the foregoing example.

Under these circumstances, the piston would traverse the last 10 feet of its range, while the pressure of the steam would vary only $3\frac{1}{2}$ pounds upon a square inch of the piston. If we bisect the angle of the cylinder by the straight line EV, it will mark out the direction of the axis of the hyperbola, and meet the curve in the vertex at V, from which we see, that if the steam were cut off at a shorter interval, the pressure during expansion would be more uniform; this is manifest from the figure, for in that case the vertex of the curve would fall lower down on the line EV, while the ordinate La would become proportionably shorter, so that the curve would come nearer to a parallelism with its asymptotes. This however is only another view of the principle alluded to above, namely, that the higher the degree of expansion, the more uniform does the pressure of the steam become, that is, the less is the difference of the variations which take place in the pressure during expansion.

If we now suppose the length of stroke to be 12 feet instead of 10 as in the above case, with the initial pressure and cut-off the same as before; then the steam in passing through the engine, will dilate itself into six times the space which it occupies under full pressure, and in that case the terminal force will be 5.83 pounds per square inch, for $\frac{35}{6}=5.83$. With this length of cylinder the index of expansion is 6, of which the hyperbolic logarithm increased by unity, is 2.79176, and this multiplied by the terminal pressure gives 2.79176 × 5.83 = 16.283 pounds per square inch for the mean pressure of the steam in passing through the engine; but we have already seen that with a cylinder of 10 feet in length, the mean pressure of the steam is 18.266 pounds upon the square inch; from which it appears, that all other things remaining the same, an engine with a cylinder of 10 feet in length is more powerful than one with a cylinder of 12 feet; so that considered theoretically, the power of an engine is diminished by lengthening its stroke, the velocity of the piston undergoing no change; but if the engine be constrained to make the same number of strokes in the same time, then the advantage will lie on the side of the longer cylinder; for in that case the velocity of the piston will be increased in a higher ratio than the mean pressure of the steam is diminished.

Again, let us suppose the length of the cylinder to be only 8 feet, the initial pressure of the steam and the point of cut-off being the same as before; then the steam will dilate itself into four times the space which it occupies under full pressure, and the terminal force of the steam will be 8.75 pounds upon the square inch of the piston.

In this case the index of expansion is only 4, and the hyperbolic logarithm of 4 increased by unity, is 2.3863, which being multiplied by 8.75 the terminal pressure, gives 20.88 pounds per square inch for the mean pressure, exerted by the steam in passing through the engine. The mean pressure in the three different cases which we have here selected, is therefore as follows.

1. For a cylinder 8 feet in length, the mean pressure of the steam is 20.88 pounds.

2. For a cylinder 10 feet in length, the mean pressure of the steam is 18.266 pounds.

3. For a cylinder 12 feet in length, the mean pressure of the steam is 16.283 pounds.

And by comparing these results with each other, we find that for a difference of two feet in the length of the cylinders, the corresponding differences in the mean pressures, are 2.614 and 1.983 pounds upon the square inch respectively.

From these results we learn, that with the same initial pressure of the steam; the same quantity of steam, or the same point of cut-off; the same diameter of cylinder, and the same velocity of piston, the shorter the stroke the more powerful is the engine. This is the correct theoretical view of the subject, but in practice circumstances may arise, of which theory is not cognizant, and which may so far modify the action of the engine and the machinery connected with it, as to give the advantage in favour of the longer cylinder, especially when retained within proper limits; and this has been affirmed by practical engineers actually to be the case; but however this may be in reality, it is obvious that the foregoing theoretical results, are decidedly in favour of the shorter cylinder, all other conditions being the same.

It is very manifest from the contemplation of the steam engine, that there must be certain proportions between the length and the diameter of the cylinder, and also between the length of the cylinder and the velocity of the piston, so that a given quantity of steam may produce the greatest quantity of useful effect. These proportions can be determined theoretically, but it does not appear from the practice of the best manufacturers, that there is any settled rule for the proportions of the cylinder, when the length of the stroke is not limited by the circumstances of construction.

The topic of greatest importance which renders it necessary to pay attention to the proportions of the cylinder, is the quantity of cooling surface to which the steam is exposed during its action. This surface ought to be the least possible, seeing that it exercises a very considerable influence in condensing the steam, and thereby very materially diminishing its power to produce motion.

The quantity of surface with which the steam comes into contact during its action, consists of one end of the cylinder, its concave surface, and one side of the piston or that on which the steam presses, which in reality is equal to the whole interior surface of the cylinder. But we must here remark, that since the concavity is only gradually brought in contact with the steam, its effect will be equivalent to about half an equal quantity of surface bounding the steam during the stroke.

Now it is obvious that the power of the engine must be a maximum, when the effect of a given quantity or weight of steam is a maximum. The problem is therefore reduced to the determination of the least quantity of surface, that is capable of containing a given quantity of steam during its action.

When the length of a cylinder is double its diameter, a given quantity of steam is contained by the least possible quantity of surface during its action in the cylinder; consequently, when the length of the stroke is not limited by the circumstances of construction and situation, it may be inferred, that the proportion of two in length to one in diameter, is the best that can be employed for a steam engine. Different makers adopt different proportions, without assigning any particular reason for their choice; but that which we have given above is derived directly from theory, and it may be satisfactory to show the method of its derivation.

By the principles of mensuration, the capacity of the cylinder is expressed by the square of the diameter multiplied by the length, and again by the constant decimal 0.7854, or the number which represents the area of a circle when the diameter is unity. The capacity therefore is represented as below, viz.:

$$capacity = 0.7854 \times diameter^2 \times length.$$

In like manner, the areas of the two ends, are expressed each, by the square of the diameter multiplied by the constant decimal 0.7854; and consequently, when taken together, both ends will be equal to the square of the diameter multiplied by the double of 0.7854 or 1.5708.

Now, as the concave surface of a right cylinder, is equal to twice the capacity divided by the diameter; therefore, the areas of the two ends of cylinder, together with half the concave surface, must be the least possible; that is,

$$diameter^2 \times 1.5708 + 2.\, capacity \div diameter = \text{a } minimum\ ;$$

and this by the fluxional analysis is found to take place when the length of the cylinder is equal to twice its diameter. This is the proportion which was determined by Mr. Blake at an early period of steam engine improvements, and we believe it is in a great measure followed by the more skilful class of manufacturers, when the circumstances of construction will admit of its adoption.

But to return to the subject of expansion, we may here observe, that the method which we have adopted of considering the subject by a reference to a curve, renders it interestingly intelligible, and since the curve by which the law of expansion is represented, is one of very easy delineation, it ought to be adopted in all cases where the nature and laws of expansion constitute the subject of inquiry.

In order, therefore, to extend the comparison between the long and the short cylinder engines, let us take another view of the question, before we decide which of them is likely to be attended with the greatest advantage when working expansively; and for this purpose, we shall consider the terminal pressure and the cut-off to be the same for all the three cylinders exemplified in the previous comparison.

Now in the case of the 10 feet cylinder, the terminal pressure of the steam is 7 pounds upon a square inch of the piston, and the steam from the boiler is intercepted when the piston has traversed 2 feet of its range. These data being referred to the 12 feet cylinder, will give 6 for the index of expansion, and 42 pounds for the initial pressure of the steam, or that under which it enters the cylinder from the boiler; whereas in the case of the 10 feet cylinder, the initial was only 35 pounds.

Computing the mean pressure of the 12 feet cylinder by the rule to the general equation (B) or (C), we find it as below, viz.

$$\textit{Mean pressure} = 7\ (\text{hyp. log. } 6 + 1) = 7 \times 2.7917595 = 19.542 \text{ pounds.}$$

This exceeds the mean pressure for the 10 feet cylinder by 1.276 pounds per square inch, for in that case, the mean pressure computed by the same rule as above, is only 18.266 pounds, and this gives $19.542 - 18.266 = 1.276$ pounds of difference on one square inch of the piston. The initial pressure, the terminal pressure, and the mean pressure in the two cases, are therefore as below.

	Initial press.	Terminal press.	Mean press.
The 10 feet cylinder	35	7	18.266
The 12 feet cylinder	42	7	19.542

It therefore appears, that for a difference of 2 feet in the length of the cylinder, and a difference of 7 pounds per square inch in the initial pressure of the steam, we obtain an addition of something more than $1\frac{1}{4}$ pounds upon a square inch of the piston.

Consequently, if there be no loss, or no great extra expense incurred, in obtaining a higher tension of the steam, or in raising it from a pressure of 35 pounds per square inch to 42 pounds, it is manifest, that if the velocity of the piston be the same in both cases, the long stroke cylinder will possess a decided advantage, an advantage which will be more strikingly displayed in comparing the absolute power of the engine, or by finding the momentum of the mean pressure for the 10 and for the 12 feet cylinders. These momenta it is obvious, will be in the proportion of 18.266 to 19.542, as we have found the mean pressure to be, by the application of the general rule, or that deduced from the equation (B) or (C).

Again, by making a similar comparison with the 8 feet cylinder, we find the index of expansion to be 4, and the initial pressure of the steam 28 pounds upon the square inch; the mean pressure is therefore as beneath,

$$\textit{mean pressure} = 7\ (\text{hyp. log.}\ 4 + 1) = 7 \times 2.3862944 = 16.704 \text{ pounds.}$$

Here the mean pressure is less than in the case of the 10 feet cylinder, and since the velocity of the piston is the same in both, the power of the engine will be less in the ratio of the mean pressure. The initial pressure, the terminal pressure, and the mean pressure of the steam for the 10 feet and the eight feet cylinders, are respectively as below.

	Initial press.	Terminal press.	Mean press.
The 10 feet cylinder	35	7	18.266
The 8 feet cylinder	28	7	16.704

It therefore appears, that although the difference between the initial pressures is here the same as in the preceding case, yet the difference between the mean pressure is somewhat greater; from which we infer, that the longer the cylinders are, the less is the difference of power for the same increase in the length, when the point of cut-off and the terminal pressures remain unaltered; this is evident from the rapid decrease in the differences of the hyperbolic logarithms, for any small increase in their corresponding numbers. There is still however another point of view, under which this question may be examined, and another comparison made between the long and the short stroke engines; and that is, by altering the point of cut-off, while the initial and terminal pressures remain unaltered in all the three cases.

It will be here observed, that more steam will be required for the long than for the short cylinders, so that it may expand from the same initial to the same terminal pressure; but the increased quantity of steam required, will only be in proportion to the increased length of the cylinders. Now, since the index of expansion in all the three cases is 5, the point of cut-off for the 8, 10 and 12 feet cylinders will be as below.

For the 8 feet cylinder, it is $\frac{8}{5} = 1.6$ feet of the stroke traversed.

For the 10 feet cylinder, it is $\frac{10}{5} = 2$ feet of the stroke traversed.

For the 12 feet cylinder, it is $\frac{12}{5} = 2.4$ feet of the stroke traversed.

In this view of the subject, the advantage is decidedly in favour of the short cylinder arrangement, since by the supposition the actual power of the engine is the same in all, as it must be when the mean pressure and the velocity of the piston are the same.

Now, from what has been shown above, it appears that in the case of the 10 feet cylinder, when the initial and terminal pressures of the steam are respectively 35 and 7 pounds upon the square inch, the point of cut-off is at 2 feet of the length of the stroke, and during these 2 feet, the steam is acting under full pressure, and during the other 8 feet it expands itself so as to terminate with a pressure of 7 pounds; in this case therefore, 2 feet of steam has been expended in completing the stroke.

Again, in the case of the 12 feet cylinder, when the initial and the terminal pressures are respectively 35 and 7 pounds to the square inch, the point of cut-off is at 2.4 feet of the length of the stroke, during which the steam acts at full pressure, and during the other 9.6 feet, it expands itself so as to occupy the entire cylinder, and terminates with a pressure of 7 pounds; and consequently 2.4 feet of steam has been expended in completing the stroke, although the actual power of the engine is no greater than before.

And lastly, in the case of the 8 feet cylinder, when the initial and terminal pressures of the steam are respectively 35 and 7 pounds upon the square inch, the point of cut-off is at 1.6 feet of the length of the stroke, during which the steam acts at full pressure, and for the remaining 6.4 feet it expands itself so as to occupy the entire cylinder, and terminates with a pressure of 7 pounds upon the square inch ; in this case therefore, there has only been expended 1.6 feet of steam, which is four-tenths of a foot less than the expenditure in the first case, and eight-tenths less than the expenditure in the second; so that in the 8 feet cylinder the same power is obtained with a less expenditure of steam.

It will now be manifest, that under each of the three aspects in which we have examined the question, there are certain conditions and circumstances of construction, that must determine to which arrangement the preference belongs; but these conditions do not legitimately come under the denomination of pure theory, and consequently, their selection must be left to the judgment of the constructive engineer. The practical reader however, will find in what we have done, much to edify, and much also, if carefully studied, that will tend to enlarge his views on the very interesting and important subject of expansion.

The foregoing calculations in reference to the double-acting expansive engine, have been conducted on the supposition, that everything as regards the construction of the apparatus, and the action of the steam, is in the most perfect condition possible, and that there is no resistance on the opposite of the piston arising from imperfect condensation; this however, can very seldom if ever be the case, for owing to the imperfections of workmanship and other causes, the full effect of the steam can never be attained, and in no instance whatever, can the entire mass of steam be wholly condensed so as to obtain a perfect vacuum. Indeed, in the actual working of a steam engine, it would only lead to disappointment were we to expect a higher indication on the vacuum gauge, than from 26 to 28 inches, or at a medium, about 27 inches of the mercurial column; the complement of this, or the defect from 30 inches, must therefore be a resistance to the motion, and since this resistance acts on the opposite side of the piston, to that on which the steam from the boiler acts, it must be applied negatively, or by subtraction, to give the true mean pressure by which the absolute power of the engine is to be calculated.

Let the symbol m denote the number of inches of mercury, corresponding to any state of the vacuum gauge, the complete vacuum being represented by 30 inches; then, if c be put to denote the correction in pounds per square inch, which becomes necessary in consequence of the imperfect condensation, we get

$$c = \frac{425}{864}(30 - m).$$

This is the quantity which has to be subtracted from the mean pressure as calculated by the general rule, to give the corrected mean pressure for the imperfect vacuum, the co-efficient $\frac{425}{864}$, being in the following manner; viz.

$$1728 \times 16 : 13600 :: 1 : \frac{13600}{27648} = \frac{425}{864}.$$

Therefore, by subtracting the above correction from the mean pressure as exhibited in equation (B), we get

$$p = \frac{\text{P}}{n}(\text{hyp. log. } n + 1) - \frac{425}{864}(30 - m),$$

and by carrying out the reduction it becomes

$$p = \{864\,\text{P}\,(\text{hyp. log. } n + 1) - 425\,n\,(30 - m)\} \div 864\,n$$

Now this expression, which involves the correction for imperfect condensation, although it appears to be a little more complicated in consequence of the correction being introduced, yet it is sufficiently easy of application; it does not however admit of a very explicit arrangement in a specific form; but the practical rule for its reduction, may be rendered intelligible by separating it into two parts as follows.

> RULE.—*Calculate the mean pressure of the steam as it is developed in passing through the engine, by the rule appropriatcd to that purpose, and reserve the result.*
>
> *From* 30 *inches of mercury which is the height due to a perfect vacuum, subtract the number of inches indicated by the vacuum gauge; multiply the remainder by* 425, *and divide the product by* 864; *then subtract the quotient from the reserved mean pressure, and the remainder will be the pressure corrected for imperfect condensation.*

It may be proper here to remark, that the rule as it is now expressed, applies to the first of the preceding equations, and the following example is worked out by the rule as it is here expressed.

Example.—Suppose the initial pressure of the steam, as it enters the cylinder, to be equal to 40 pounds upon a square inch of the piston, and let the steam from the boiler be cut off, when the piston has traversed one-fourth of its range, what will be the mean pressure of the steam in passing through the cylinder, supposing the vacuum gauge to indicate only 26 inches.

Since the steam from the boiler is intercepted at one-fourth the length of the stroke, the index of expansion is 4, and because the initial pressure of the steam is 40 pounds upon the square inch, the terminal pressure must be 10 pounds; and consequently, the mean pressure uncorrected, is as underneath, viz.

$$\textit{mean pressure} = 10\ (\text{hyp. log. } 4+1) = 10 \times 2.38629 = 23.8629 \text{ lbs.}$$

But by the question, the vacuum gauge is found to indicate a rise of only 26 inches in the mercurial column, and this is equivalent to a resistance of 4 inches given out by the uncondensed steam; hence by the above rule, we get

$$\textit{correction} = \frac{425}{864}(30-26 = \frac{425 \times 4}{864} = \frac{425}{216} = 1.9676 \text{ lbs.}$$

Therefore, by subtracting this result from the mean pressure previously calculated, the corrected mean pressure becomes

$$23.8629 - 1.9676 = 21.8953 \text{ pounds per square inch.}$$

It must be observed however, that from the result just obtained, all the other resistances to which the working of a steam engine is liable, must be deducted; but as these can only be determined by observation when the engine is in action, it would answer no useful purpose, to embarrass the formula with a multiplicity of symbols, for representing their effects or estimating their magnitude.

Another correction becomes necessary in consequence of the clearance of the cylinder; the effect of this is of a contrary nature to that which we have just discussed, and will therefore tend to lesson its amount; but since the additional power derived in this way, is very small in respect of the quantity of steam expended, it would be of little use to include its effects, when the influence of other and greater resistances is perfectly known.

SECTION VI.

THE DOUBLE-ACTING NON-CONDENSING ENGINE, WORKING BY EXPANSION.

IN the double-acting non-condensing engine working by expansion, we have a constant resistance acting on that side of the piston, opposite to the side on which the steam from the boiler exerts itself; but since the quantity of that resistance is known, we have only to deduct its amount from the mean pressure as calculated by the rule to the general equation (B) or (C), and the remainder thus obtained, will be the corrected mean pressure to be employed in computing the power of the engine in the usual way.

In this kind of expansive engine, we are under the necessity of generating the steam under a very high temperature, in order to insure a sufficient terminal force for carrying the piston to the end of the stroke; for since the atmosphere operates against the expansive force of the steam, it is manifest that the terminal force must be of sufficient intensity to overcome the atmospheric resistance, together with that arising from friction and other causes, and moreover, retain a competent surplus of power to keep the engine in motion; for if these conditions be not satisfied, a stoppage of the engine will be the result, unless the machinery has acquired a sufficient momentum, to carry the piston onwards to the end of the stroke, or the point of return.

It would however be injudicious to rely on this accumulated momentum, and it is obvious, that whatever the terminal pressure falls short of the aggregate resistances, must be lost power; for this reason therefore, it is necessary that the terminal force should always be in excess by so much, as shall be capable of overcoming the resistance of the atmosphere and friction of the parts.

To assign a general limit to the terminal pressure of the steam, or to state a point beyond which the expansion ought not to be carried, constitutes a problem which theory alone cannot resolve in every case, since so much depends upon the effective power of the engine, and the nature of the work to which it is applied. But when the conditions of working and the required power are known, analysis will in most cases, guide us to a proper limit for the terminal pressure; and from that, the point of cut-off and also the initial pressure of the steam may readily be determined, so as to satisfy any proposed conditions, respecting the construction and proper working of the engine.

The pressure of the atmosphere is generally reckoned at 15 pounds upon a square inch of surface; but at a medium state of density and temperature, it is something less, being nearly equal to 14.75694 pounds; for we have,

$$34 \times 62.5 \div 144 = 14.75694, \text{ the pressure per square inch.}$$

Consequently, in all our calculations respecting the non-condensing engine, when working by expansion, this constant resistance to the action of the steam must be taken into account, together with the resistance arising from friction and other causes.

In order therefore to simplify the application of this correction generally, it becomes necessary to represent it by a specific symbol, and there is nothing more appropriate for the purpose than the letter r, that being the initial letter of the word resistance itself.

Now in the case of the general equation (B), which represents the mean force or pressure evolved by the steam in passing through the engine; that is, the force which, if exerted uniformly throughout the stroke, would produce the same effect, as is actually produced by the variable pressure; it is evident, that the resistance of the atmosphere must always be deducted, in order to obtain the mean pressure that becomes effective in working the engine, when further corrected for the loss by friction and other causes. The equation for the mean pressure when thus corrected, is as below, viz.

$$p = \frac{P}{n}(\text{hyp. log. } n + 1) - r.$$

From this expression it is manifest, that when we have the initial pressure, and wish to find the index of expansion, so as to obtain a given mean pressure, this proposed pressure must always be augmented by that of the atmosphere.

Let this be the problem which is now required to be resolved, and the above equation transposed for that purpose, and adapted to common logarithms, becomes

$$\frac{1}{n}(\log. n+0.4342945,=\frac{0.4342945\,(p+r)}{\mathrm{P}}$$

The reduction of this equation in all cases of practice, is sufficiently easy, but since it requires the application of one or other of the approximating rules to be found in books of algebra, it becomes rather difficult to describe the operation in words; but to those who are conversant with the rules of position, any particular case may be resolved with the greatest facility.

In order therefore to effect the reduction of the above equation, in which n is the quantity to be determined, we have only to assume such a value of the thing required, as by adding to its common logarithm the constant quantity 0.4342945, and dividing the sum by the assumed value, the quotient shall be equal to the sum of the mean and atmospheric pressures, multiplied by 0.4342945, and divided by the given initial pressure, or that under which the steam enters the cylinder and operates upon the piston.

In order therefore, to show the mode of reducing the equation, we propose the following numerical example, in which, if the process be carefully considered, the resolution of any other case of a similar character will be a matter of the greatest simplicity.

Example.—Suppose the initial pressure of the steam to be 75 pounds upon the square inch; what must be the relative expansion of the steam, or in other words, how often must it expand itself from the bulk under full pressure, to give a mean pressure of 37.75 pounds upon a square inch?

In this example, it is required to determine what part of the stroke the piston must travel over, before the flow of steam from the boiler is intercepted, in order to obtain a given mean pressure, which shall also include the atmospheric resistance, at its medium state of 14.75 pounds upon a square inch.

Let the given numerical values of the several quantities P, p and r, be substituted instead of them in the foregoing equation for the value of n, and it becomes as follows, viz.

$$\frac{1}{n}(\log.\ n+0.4342945)=\frac{0.4342945\,(37.75+14.75,}{75}$$

and if this expression be reduced to its simplest form, it is

$$\frac{1}{n}(\log.\ n+0.4342945)=0.3040062.$$

Having reduced the expression to this very simple form, it is perceived at a single glance, that the value of the unknown quantity n, cannot be greater than 3, nor less than 2; indeed it is almost evident, that the real value differs from 3 by a very small quantity; for the common logarithm of the number 3, is 0.4771213, consequently, according to the reduced expression, it is

$$\tfrac{1}{3}(0.4771213+0.4342925)=0.3038053.$$

Now this result differs from the absolute given quantity 0.3040062, by the minute fraction 0.0002009; for by subtraction it is $0.3040062-0.3038053 =0.0002009$, the error in defect, a condition which indicates, that the assumed value is rather in excess of the true value, because the differences of the logarithms increase as the numbers decrease.

Since the resulting number on the supposition of 3 being the value sought, differs so very little from that which is given by the question, it is probable that in any practical case, the assumed value of n would be considered as the true value, and accordingly the point of cut-off would be taken at one-third of the length of the stroke; but in some very delicate constructions, it may be necessary to approximate more nearly to the value of the thing required to be found.

Now, we find that by substituting 75 for P the initial pressure, and 3 for n the index of expansion, the mean pressure when computed by the appropriate rule, comes out equal to 52.46531 pounds per square inch; for the hyperbolic logarithm of 3 is 1.0986124; hence we have

$$\textit{mean pressure}=25\,(1.0986124+1)=52.46531.$$

And subtracting from this the mean resistance of the atmosphere, we obtain $52.46531-14.75=37.71531$ pounds for the mean effective pressure of the steam. But by the question it ought to be 37.75 pounds; we must therefore endeavour to find a more correct value for the quantity n, and a single trial will show on what side the supposition is to be made; but we have already stated that n cannot exceed 3, we must therefore make the supposition a little less, and by supposing $n=2.99$, we shall probably obtain an error in excess, which will limit the true value between 3 and 2.99, or less than 3 and greater than 2.99.

Let us therefore make trial with 2.99, since we are certain that the true value is very little less than 3, and we have

$$\log.\ 2.99+0.4342945=0.4756712+0.4342945=0.9099657,$$

and when this is divided by 2.99 the assumed number, we get

$$0.9099657 \div 2.99=0.3043368.$$

In this case therefore, the result is 0.0003306 greater than the given quantity, from which we know that the assumed value is less than just, we consequently have the errors of different kinds, and for this case the rule of double position gives the following process for the corrected value of n.

First assumed value, $3 \times 3306=9918$, product,
Second assumed value, $2.99 \times 2009=6006.91$, product.

Here then the sum of the products, is $9918+6006.91=15924.91$; and the sum of the errors, is $3306+2009=5315$; therefore by division we get

$$n=15924.91 \div 5315=2.9962 \text{ very nearly.}$$

The value of n which we have now obtained, if substituted in the expression representing its value, will give

$$\frac{1}{2.9962}(\log.\ 2.9962+0.4342945)=3040062, \text{ as required.}$$

Now, the logarithm of 2.9962, is 0.4765708, to which let the constant 0.4342945 be added, and we get $0.4765708+0.4342945=0.9108653$, and dividing this sum by 2.9962, gives $0.9108653 \div 2.9962=0.3040062$ as it ought to be according to the requirement of the problem.

The hyperbolic logarithm of 2.9962, is 1.0973447, which being increased by unity and multiplied by $\frac{75}{2.9962}$, gives 52.5 pounds for the mean pressure including the resistance of the atmosphere; from which let 14.75 be subtracted, and we get

$$52.50-14.75=37.75 \text{ pounds per square inch.}$$

In this way therefore, may the index of expansion be found, in order to obtain any mean pressure whatever; care must however be taken, that the power of the boiler is adequate to generate steam of sufficient intensity to admit of the proposed pressure, together with all the necessary deductions for different resistances which have to be met from different sources.

The preceding is the method of computing the point of cut-off, when the initial and mean pressures of the steam are given, the latter including the resistance of the atmosphere; but when the initial and terminal pressures of the steam are given under the like conditions, the calculation becomes greatly simplified; for in that case it would be:—

> *As the initial pressure is to unity, so is the terminal pressure, increased by the resistance of the atmosphere in its medium state of density, to the part of the stroke traversed by the piston before the steam is cut off.*

Example.—The initial pressure of the steam as it enters the cylinder, is 75 pounds upon the square inch; at what point of the stroke must the steam be cut off, so that the terminal pressure may be 25.03 pounds per square inch?

Here we have $25.03 + 14.75 = 39.78$; then by the above analogy, it is

$$75 : 1 :: 39.78 : 0.53,$$ the point required.

It would be quite superfluous to dwell longer on the single cylinder non-condensing engine, as the several calculations would be the same as for the condensing engine, with the exception of the necessary allowance for the atmospheric resistance, we shall therefore proceed to consider the different forms of the combined cylinder engine, in which the principle of expansion admits of a wider range.

SECTION VII.

THE COMBINED CYLINDER EXPANSIVE ENGINE.

THE subject of expansion as we have treated it in the preceding department of the work, has reference to the operation and effects of steam when a certain quantiry or weight of it is expanded in the same vessel into which it is originally admitted from the boiler, the vessel being supposed to be of uniform diameter throughout the whole of its length.

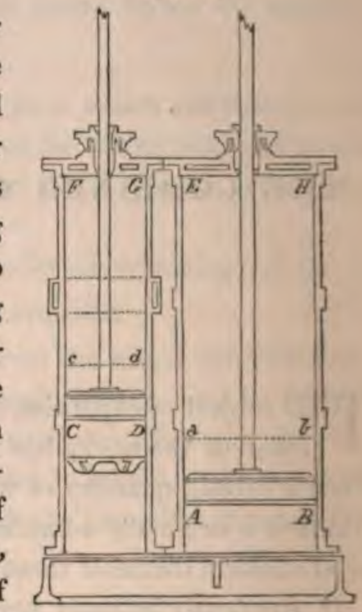
It frequently happens however, that the expansion is allowed to take place in a separate vessel of three or four times the capacity of that into which the steam is at first received, these vessels having a passage of communication between them, which can be opened and shut at pleasure, as the peculiar circumstances of the case may require.

It also sometimes happens, that the steam is expanded in both vessels, and when that is the case, the calculations are rendered a little more complex, in consequence of a subsidiary operation becoming necessary for obtaining the mean pressure of the steam in the first vessel; but notwithstanding this, the process is very easily performed, either by the finite expression as derived from analysis, or by the method of approximation as we have elsewhere exemplified it.

This arrangement of the vessels or cylinders in which the steam acts, is that which takes place in the combined cylinder engine when constructed according to Woolf's principle; where the cylinders are placed side by side, in such a manner, that the steam is permitted to act simultaneously on the same side of both pistons; that is, on either side of the smaller piston under full pressure as it is received from the boiler, and on the corresponding side of the larger piston under a pressure due to the degree of expansion which obtains at any specified point of the movement; the steam being permitted to flow from the smaller cylinder where it acts with all its force, and expands itself into the larger cylinder where it acts with a diminished force.

The accompanying drawing *fig.* 11, represents the two vessels or cylinders placed under the specified conditions, where it will readily be understood, that while the steam is admitted from the boiler into the lower part of the lesser cylinder under full pressure, the steam which was previously admitted, is at the same time passing from the upper part of the lesser cylinder, into the lower part of the larger one, and there acting on the under side of the greater piston by expansion, a partial vacuum being formed on the other side, by means of condensation effected in the usual way, the temperature of the uncondensed steam being usually about 120 degrees of Fahrenheit's thermometer. It therefore appears, that the steam is acting on the under side of both pistons at the same time, but with very different degrees of intensity, the pressure exerted on the lesser piston being constantly the same, while that on the greater one is continually varying between limits that depend upon the relative capacities of the cylinders, and the intensity of the initial pressure, or that which the steam exerts on the smaller piston, before it is admitted into the larger cylinder to act by expansion.

Fig. 11.



We have here detailed the operation that takes place, and the phases that display themselves when the pistons are performing their upward stroke; but it is easy to perceive, that precisely the same things will occur when they are making their downward stroke, and so on for every reciprocation of the pistons, as long as the steam is supplied from the boiler to continue the action of the engine.

It has been stated above, that steam from the boiler is admitted into the lesser cylinder under a constant pressure, and that it exerts a constant degree of force upon the lesser piston; this is certainly the case, but although the pressure with which the steam acts upon the piston is constantly the same, yet the force which it exerts to put the piston in motion is variable throughout its range, increasing gradually from the instant it first enters the cylinder, until it attains its full effect at the termination of the stroke, so that the steam is acting with full pressure at the time it begins to enter the larger cylinder and operate by expansion.

But on the other hand, the steam which flows from the lesser cylinder into the greater, and there dilates itself, exerts its greatest force to move the piston at the instant of impact, which force continually decreases until the termination of the stroke, being precisely the reverse of what takes place in the lesser cylinder; for there the effect of the steam to move the piston continually increases, from the time it enters the cylinder until the stroke is completed.

Here then it is obvious, that in this arrangement, the force of the steam as it acts on both pistons jointly, constitutes the moving power, and consequently it is never less than the force of steam on the smaller piston, or the force under which it enters the lesser cylinder from the boiler.

All this is manifest, for when the pistons are at or near the lowest point of their respective ranges, as represented in the drawing, *fig.* 11, and the steam admitted from the boiler under the lesser piston, this steam presses on the under side of the piston, with a force that is due to its density and temperature; but this force is counteracted by the pressure of the steam between the pistons, which acts on the upper side of the lesser piston in opposition to the full pressure steam from the boiler, with a force due to the degree of expansion it has undergone, in passing into the larger cylinder, and working against a vacuum more or less perfect. But whatever may be the effect of this counteraction on the lesser piston, it is again balanced by an equal pressure on an equal area of the larger piston; so that the two pressures together, produce the same effect in moving the engine, as if the steam from the boiler were not counteracted at all by that between the pistons.

In calculating the effect of expansion according to this arrangement of the steam cylinders, it becomes necessary to know their relative capacities; for since they are different both in length and diameter, the capacities can only be compared by the squares of the diameters drawn into the lengths; and when great accuracy is required, it will also be necessary to know the capacity of the spaces or passages between the cylinders. To compute the capacities of the cylinders :—

Put $D = AB$, the diameter of the larger cylinder, in which the expansion takes place;
$d = CD$, the diameter of the smaller cylinder, in which the steam acts with full force;
$l =$ the length of the larger cylinder, in which the steam acts by expansion;
and $l' =$ the length of the smaller cylinder, in which the steam acts at full pressure.

Then, by the rules of mensuration, *the capacities of the cylinders are in the compound ratio of their lengths and the squares of their diameters.* Therefore, if the symbols c and c', be put to denote the capacities of the larger and smaller cylinders respectively, we have

$$c : c' :: lD^2 : l'd^2.$$

Consequently, the ratio of the capacities as we have here represented them, or the number of times that the larger cylinder contains the lesser, which is also the number of times that the steam expands itself, must be represented as below, viz.

$$n = \frac{c}{c'} = \frac{l D^2}{l'd^2};$$

in which expression the symbol n, is what we have hitherto denominated the index of expansion.

Now, in estimating the effect of expansion in this arrangement, it will be sufficient for the purpose, to trace the operation for one stroke of the piston only, since precisely the same things occur for every other stroke, whether it be ascending or descending, and for this reason, the details of one operation will be found sufficient to establish the theory, and need not be repeated.

By examining the construction represented in the drawing, *fig.* 11, it will readily be perceived from the connexion of the several parts, that both the pistons must ascend and descend simultaneously, and by doing so, they complete their strokes at one and the same instant of time; but it is manifest from the arrangement, that their velocities of motion are not the same, for the spaces through which they respectively travel at each stroke, are proportional to the lengths of the levers on which they act, or which is the same thing, the distances from the centre of the beam, of the points to which the piston rods are applied; and during the motion, the lesser piston is acted upon by steam received directly from the boiler, while the larger one is acted upon by the expanding power of the steam which previously occupied the lesser cylinder, or that which was received from the boiler during the preceding stroke of the engine.

The principle of operation, in so far as regards the admission of the steam, its eduction and condensation, being perfectly well understood by practical men in general, it is needless for us to dwell on these particulars in this place, we shall therefore proceed to compute the effect of expansion as developed in the larger cylinder; which effect being wholly additional to that produced by the steam, when acting at full pressure throughout the stroke of the lesser piston, confers on this species of engine, all the superiority that has been attributed to it.

There are various methods of calculating the power of an engine of this sort, but in whatever way the mechanical effect is determined, if the operation be rightly performed, the results ought to be very nearly the same, and in order to give a full and explicit elucidation of the subject, we shall calculate the power of a combined cylinder engine:—

1. *By finding the mean pressure on each of the pistons, and computing their joint effects according to their respective areas and velocities.*
2. *By supposing the mean pressure to be collected on the lesser piston, and computing its effect according to the area and velocity.*
3. *By supposing the lesser piston to act under full pressure, and the larger one by expansion, or the mean pressure on its surplus area.*

And for the purpose of comparing the three methods with each other, it will be convenient to select for an example, some engine of this particular class which is actually in operation, and of which the performance is known.

Now, we have seen and experimented upon an engine of Woolf's arrangement, which performs its duty with the greatest regularity and precision; the cylinders of which are $15\frac{1}{2}$ and 31 inches in diameter, and the corresponding lengths are $4\frac{1}{2}$ and 6 feet, or 54 and 72 inches respectively, consequently, the capacities of the cylinders are as 3 to 16, or as 1 to $5\frac{1}{3}$, as can readily be ascertained by the rules of mensuration.

This engine in its usual working order, is estimated at from 35 to 40 horse power; but is said to be capable of doing the duty of a 60 horse engine, when working to the full extent of its power; the usual working pressure of steam in the lesser cylinder, is 35 lbs. on a square inch of the piston, and the consumption of coals is something less than three pounds per horse power per hour.

This may therefore be considered as a good example for illustrating the application of our theory, and the method of calculating the power by the first of the foregoing varieties, is as follows.

The lesser cylinder being $15\frac{1}{2}$ inches in diameter, and $4\frac{1}{2}$ feet or 54 inches in length, its capacity, according to the common rules of mensuration, is equal to the area of its base or end, multiplied by its length; but the area of the base, is equal to the square of the diameter, multiplied by the constant decimal 0.7854, which decimal represents the area of a circle whose diameter is unity; hence we have

$$15.5 \times 15.5 \times 0.7854 \times 54 = 10189.3869 \text{ cubic inches.}$$

The number of cubic inches in one cubic foot, is 1728; therefore, if the above number of cubic inches be divided by 1728, the quotient will be the number of cubic feet in the lesser cylinder; that is,

$$\frac{10189.3869}{1728} = 5.8966 \text{ cubic feet.}$$

This, therefore, is the quantity of steam consumed for every single stroke of the lesser piston, generated under a temperature due to a pressure of 35 pounds per square inch of surface, or very nearly 259 degrees of Fahrenheit's thermometer.

In like manner, since the larger cylinder is 31 inches in diameter and 6 feet or 72 inches in length, its capacity found in the same way as above, is 31.4488 cubic feet; for we have

$$31 \times 31 \times 0.7854 \times 72 = 54343.3968 \text{ cubic inches;}$$

and if this be divided by 1728, the number of cubic inches in one cubic foot, we get

$$\frac{54343.3968}{1728} = 31.4488 \text{ cubic feet.}$$

Now, if the latter of these results be divided by the former, we find that the capacities of the cylinders, are to each other as 1 to $5\frac{1}{3}$, as above stated, for we get

$$31.4488 \div 5.8966 = 5.3 \text{ or } 5\tfrac{1}{3}.$$

This ratio, however, is readily assignable without actually computing the capacities of the cylinders in the manner that we have done above; for since the one diameter is just double of the other, the areas of the pistons are to each other in the proportion of 1 to 4, and the ratio of the areas is therefore $\frac{1}{4}$. But the length of the one is 54 inches, while that of the other is 72 inches; the ratio of the lengths is therefore $\frac{54}{72}$ or $\frac{3}{4}$; but by the principles of geometry, the capacities of the cylinders are as the lengths and the squares of the diameters conjointly; hence it is

$$\frac{1}{4} \times \frac{3}{4} = \frac{3}{16}, \text{ or dividing by 3, we get } \frac{1}{5\frac{1}{3}}, \text{ the ratio as above.}$$

It therefore appears, that having got the ratio of the capacities, it is not necessary to have the absolute magnitudes themselves, as the same results will arise from the one as from the other, for since each term of the ratio is affected by the same constant quantity or decimal 0.7854, it is eliminated from both.

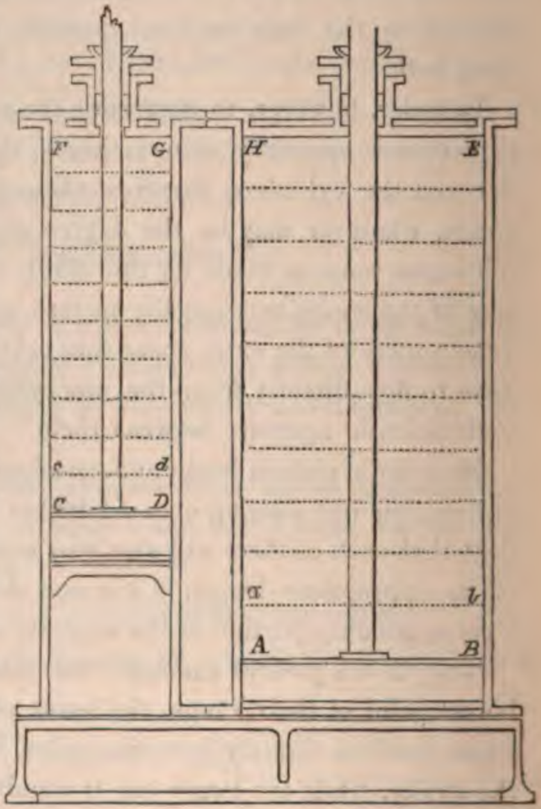
In order, however, to determine the true relative expansion of the steam, it becomes necessary also to know the capacity of the steam passages between the cylinders, for since these passages must always be filled with steam, whatever may be the degree of expansion, it is obvious, that some allowance must be made for the effect thus produced upon the mean pressure of the steam between the pistons, as this mean pressure must be less, in consequence of the extra space thus occupied, than it would be, if the steam were to flow directly from the one cylinder into the other through a mere perforation or aperture between them.

Since both pistons begin and terminate their strokes at the same instant of time, the one passing over 54 inches and the other 72 inches; it is evident that each of them will also pass over a corresponding proportional part of its appropriate length in one and the same space of time, and generate a corresponding portion of the capacity of the cylinder in which it moves.

Now, in the present example, we shall suppose the pistons to be at the lowest point of their range, the lesser piston having been pressed down by steam received directly from the boiler, and under full pressure throughout the stroke, while the larger one is simultaneously forced down by the steam being admitted above it from the lesser cylinder, and acting by expansion through the space comprehended between the pistons.

In this state of things, it is manifest, that the space above the lesser piston considered in any position, is filled with steam directly from the boiler, while that above the larger one in the same position, is filled with steam dilated in the proportion of $5\frac{1}{3}$ to 1; but when the stroke is completed as represented in the following drawing, *fig.* 12, the steam above the larger piston is actually dilated $5\frac{1}{3}$ times, and consequently it retains only three-sixteenths of the force which it exerted on the lesser piston; then, if a communication be opened between the upper side of the larger piston and the condenser, while another is opened between the under side of the lesser piston and the boiler, and also between the upper side and the lesser piston and the under side of the greater, the steam which is above the larger piston will be condensed and form a vacuum, while that which is above the lesser piston will flow under the greater, and raise it by expansion, the lesser piston being raised by steam direct from the boiler under full pressure.

Fig. 12.



Let us now suppose that each of the pistons is elevated through one-tenth part of its range, as represented by the dotted line *ab* and *cd*, *fig.* 12, a communication being then open between the upper side of the lesser piston, and the under side of the larger one; then the steam which before occupied only the space CDGF in the smaller cylinder, will now occupy the space *cd*GF, together with the space AB*ba*, and the passages of communication; that is, the whole space between the pistons. But the space AB*ba* is greater than the space CD*dc*, to which add the steam passages, and it will be seen that the steam has expanded itself into a larger space, and in consequence of that expansion, has lost a proportional part of its elastic force. Now the entire space CDGF, or the capacity of the smaller cylinder, is represented by the number 3, while the space ABHE, or the capacity of the larger cylinder, is represented by 16. Consequently, if each of the cylinders be divided into ten equal parts, as represented by the dotted lines in the cylinders, the capacity of each part in the lesser cylinder, will be

represented by 0.3, and in the larger one by 1.6; these being the tenth-parts of the respective cylinders. It will however, be better to take the tenth-part of their respective capacities in cubic feet, for in that case we can add the capacity of the steam passages in the same measure, and in its actual value, so that the true expansion will thus be accurately determined. Now, the cubic contents of the smaller cylinder is 5.900 feet, and that of the larger one 31.450 feet, and consequently, their tenth-parts are 0.590 and 3.145 cubic feet respectively; and if we estimate the steam passage between the cylinders at half a cubic foot, the entire space between the pistons when they have traversed one-tenth of the stroke, is
$5.900-0.590+3.145+0.500=8.955$ cubic feet, the first term of the series.

This is therefore the first term of the series, for which the elastic force or pressure of the steam has to be calculated, on the supposition that each of the cylinders is divided into ten equal parts, by planes drawn parallel to their bases; or which is the same thing, when the pistons are in the positions represented by the several dotted lines in *fig.* 12 foregoing. The other terms of the series will therefore be obtained, if we continually subtract the constant decrement from the capacity of the smaller cylinder, and increase the several remainders by the successive increments of the larger cylinder, together with the capacity of the steam passages. Thus we have

$5.900-1.180+6.290+0.500=11.510$ cubic feet, second term.
$5.900-1.770+9.435+0.500=14.065$ cubic feet, third term.
$5.900-2.360+12.580+0.500=16.620$ cubic feet, fourth term.
$5.900-2.950+15.725+0.500=19.175$ cubic feet, fifth term.
$5.900-3.540+18.870+0.500=21.730$ cubic feet, sixth term.
$5.900-4.130+22.015+0.500=24.285$ cubic feet, seventh term.
$5.900-4.720+25.160+0.500=26.840$ cubic feet, eighth term.
$5.900-5.310+28.305+0.500=29.395$ cubic feet, ninth term.
$5.900-5.900+31.450+0.500=31.950$ cubic feet, tenth term.

These therefore, are the several terms for which the pressures are to be calculated; and since by the general law of elastic fluids, the elasticities are inversely as the spaces, we have the following analogies for the pressures at each tenth of the stroke.

$8.955 : 35 :: 5.9 : 23.060$ pounds per square inch.
$11.510 : 35 :: 5.9 : 17.941$ „ „ „
$14.065 : 35 :: 5.9 : 14.682$ „ „ „
$16.620 : 35 :: 5.9 : 12.425$ „ „ „

19.175 : 35 : : 5.9 : 10.769 pounds per square inch.
21.730 : 35 : : 5.9 : 9.503 „ „ „
24.285 : 35 : : 5.9 : 8.503 „ „ „
26.840 : 35 : : 5.9 : 7.694 „ „ „
29.395 : 35 : : 5.9 : 7.025 „ „ „
31.950 : 35 : : 5.9 : 6.463 „ „ „

Now these are the pressures per square inch between the pistons, at every tenth part of the stroke, and from these the mean pressure can readily be determined.

Taking the sum of these computed pressures, we get 118.065 pounds for the 10 computed spaces; but the initial pressure with which the steam acts upon the larger piston, is only 32.266 pounds per square inch, being reduced from 35 pounds to 32.266 pounds in filling up the steam passage between the cylinders and space for clearance; for it is,

$$5.9+0.5=6.4:35::5.9:32.266.$$

Therefore, by adding this initial pressure to the above sum of 118.065 pounds, we get for the entire sum of the terms 150.331, which being divided by 11 the number of terms, gives 13.67 pounds per square inch for the mean pressure between the pistons. We have however, given another method of approximating to the mean pressure of the expanded steam, and since it gives a more correct result than that which we employed above, we think proper to illustrate the method of applying it.

The initial pressure of the steam, or that which it exerts at the instant it enters the larger cylinder, is 32.266 pounds per square inch, as we have computed it by the above analogy, and the terminal pressure is 6.463 pounds; therefore by addition, we get

32.266+6.463 = 38.729 pounds, the sum of the extreme terms,
64.039 × 4 = 256.156 pounds, four times the sum of the even terms.
47.563 × 2 = 95.126 pounds, twice the sum of the odd terms.

390.011 = sum total; and dividing this sum by 30, the mean pressure becomes 13 pounds per square inch between the pistons very nearly. The divisor 30 which we have employed, is simply one-third of the common interval between the terms, on the supposition that the whole length of the cylinder is represented by unity. Thus we have $\frac{1}{3}$ of $\frac{1}{10}=\frac{1}{30}$.

By assuming a greater number of terms in the series, the mean pressure developed by the steam during expansion would be more rigorously approximated, whichsoever of these methods be applied; but without having recourse to any of the approximating methods, the true pressure can be determined directly by the rule adapted expressly for that purpose, as given in a former part of this work. The formula from which this rule is derived is $p=\frac{P}{n}\times$ hyp. log. n; but the value of n as referred to the two cylinders of the engine, is expressed by the term $\frac{lD^2}{l'd^2}$; consequently, if this value of n or the index of expansion, be substituted instead of it in the above expression for the mean pressure, we get

$$\textit{mean pressure}=\frac{l'd^2 D}{lD^2}\times \text{hyp. log. } \frac{lD^2}{l'd^2}$$

This form of equation is more convenient than that which precedes it for our present purpose, in consequence of its involving the dimensions of both the cylinders, and the practical rule for reducing it, may be expressed in words at length, as below.

> Rule.—*Multiply the initial pressure of the steam in pounds per square inch, by the capacity of the lesser cylinder, and divide the product by the capacity of the greater; then multiply the quotient by the hyperbolic logarithm of the capacity of the greater cylinder divided by the capacity of the smaller, and the product will be the mean pressure of the steam between the pistons expressed in pounds per square inch.*

In the example now under consideration, the initial pressure of the steam as it operates in the lesser cylinder, is 35 pounds upon a square inch; but when it begins to act in the larger cylinder, its elastic force is reduced to 32.266 pounds upon the square inch, in consequence of filling up the passages between the cylinders. This therefore must be considered as the initial pressure or value of P as it is to be used in the calculation.

Now, since the capacity of the lesser cylinder is represented by 3, and that of the greater one by 16, the above rule leads us to the following process, for the mean pressure of the steam between the pistons; viz.

$$\textit{mean pressure}=\frac{3\times 32.266}{4\times 4}\times \text{hyp. log. } \frac{16}{3}.$$

But the hyperbolic logarithm of $\frac{16}{3}=1.6739763$, which being multiplied by 96.798 and divided by 16, gives 10.127 pounds for the mean pressure;

now this is very considerably less than the result of either of the foregoing approximating methods; but if the effect of the initial pressure were taken into account here also, the difference would be much less than it is represented to be.

Now the initial pressure upon the lesser piston is 35 pounds per square inch, but it is counteracted on the opposite side by the mean pressure between the pistons, and this mean pressure must be subtracted to give the mean on the lesser piston, so that we have $35 - 10.127 = 24.873$ pounds per square inch.

Since the total effect of the expanded steam to communicate motion to the piston, is equal to the area of the piston, multiplied by the mean pressure of the steam in pounds per square inch, and because the momentum or mechanical effect, is equal to the motive power multiplied by its velocity, or the space which it passes over in a given time; it follows, that if we put

m = the momentum or mechanical effect of the expanded steam, estimated in the usual way,

and v = the velocity of the piston in feet per minute,

Then we have, *momentum* = 0.7854 × *diameter*^{2} × *mean press.* × *vel.*; and substituting for the mean pressure, its foregoing value in terms of the dimensions of the cylinders, we get

$$momentum = \frac{0.7854 d^2 l' v \mathrm{P}}{l} \times \text{hyp. log. } \frac{l \mathrm{D}^2}{l' d^2}$$

Now this equation for the momentum does certainly appear something complex; but notwithstanding the number of factors which it involves, its reduction is as simple as can be desired; but the practical rule will be most easily deduced from the first form of the expression, in which the entire process is distinctly indicated. It is as follows.

RULE.—*Calculate the mean pressure by expansion, as directed in the rule for that purpose; then, multiply together, the mean pressure thus found, the area of the piston in square inches, and its velocity in feet per minute, and the product of the three will be the momentum sought.*

The rule as it is here enunciated, is applicable to the larger cylinder in which the expansion takes place; but it applies with equal facility to the lesser cylinder, by substituting the words *initial pressure* for *mean pressure,* as used in the rule.

Taking therefore the velocity of the larger piston at the usual rate of 220 feet per minute, the velocity of the lesser one will be 165 feet in the same time; for it is 72 : 220 : : 54 : 165. Consequently, for the momentum of the lesser piston in pounds raised one foot high per minute, we have

$$15.5 \times 15.5 \times 0.7854 = 188.69 \text{ square inches the area;}$$

$$\text{and } 188.69 \times 24.873 \times 165 = 774400 \text{ pounds the momentum.}$$

In like manner, for the momentum of the larger piston in pounds avoirdupoise, raised one foot high per minute, we have as follows,

$31 \times 31 \times .7854 = 754.76$ square inches, the area of the piston, and $754.76 \times 10.127 \times 220 = 1681577$ pounds, the momentum or mechanical effect.

Therefore, by adding these two momenta together, and dividing the sum by 33000, the gross power of the engine becomes

$$(774400 + 1681577) \div 33000 = 74.423 \text{ horse-power;}$$

and by allowing four-tenths of this amount for the accumulated losses by friction and other causes, the effective power of the engine will be

$$74.423 \times 0.6 = 44.654 \text{ horses very nearly.}$$

If the gross power exerted by each of the cylinders be represented separately, we get as follows :—

For the gross power of the lesser cylinder, it is, $774400 \div 33000 = 23.466$ horses.

For the gross power of the larger cylinder, it is, $1681577 \div 33000 = 50.957$ horses.

And if each of these be reduced by four-tenths for the accumulated losses, the effective power of each will be as follows.

For the effective power of the lesser piston, it is, $23.466 \times 0.6 = 14.080$ horses.

For the effective power of the larger piston, it is, $50.957 \times 0.6 = 30.574$ horses.

And the sum of these being taken, gives $14.080 + 30.574 = 44.654$ horses, the same as we found above for the effective power of the engine.

If we calculate the power of the engine by the second method previously specified, viz., when the mean pressure is supposed to be collected on the lesser piston, we have the following process.

The number of times that the capacity of the lesser cylinder is contained in that of the larger, together with the steam passages between the cylinders, is 5.418, for $(31.4488+0.5)\div 5.8966=5.418$; and the hyperbolic logarithm of 5.418 is 1.6897586. Now, the rule for the mean pressure of the steam when collected on the lesser piston, is as follows.

> Rule.—*Multiply the hyperbolic logarithm of the number of times, that the capacity of the lesser piston is contained in that of the greater, by the pressure in the boiler in pounds per square inch, and the product will be the mean pressure, reduced to the surface of the lesser piston.*

Now, in the present example, the initial pressure of the steam as it enters the lesser cylinder, is 35 pounds upon the square inch, or about 46.7 pounds on the safety valve, and when it enters the larger cylinder, it is reduced to 32.266 pounds per square inch, consequently, the mean pressure when collected on the lesser piston, is $1.6897586\times 46.7=78.9$ pounds per square inch; from which the gross power of the engine is found as underneath.

Diameter of piston, 15.5 inches . .		log. 1.1903317
		2
Diameter of piston squared, 240.25 . .		log. 2.3806634
Constant for circular areas, 0.7854 . .		log. 9.8950899
Mean pressure on the lesser piston, 78.9 lbs.		log. 1.8970770
Velocity of the piston, 165 feet . .		log. 2.2174839
Constant for horse power, 33000 .	ar. co.	log. 5.4814861
Gross power of the engine = 74.439 horses		log. 1.8718003

Here then we have obtained nearly the same gross power, as by the pr ceding method, and taking the accumulated resistances at the same amou the effective power will be $74.439\times 0.6=44.66$ horses.

If the power of the engine be performed according to the third meth where the steam is supposed to act with full force on the lesser pis throughout the stroke, the operation will be as follows, viz.

Diameter of piston, 15.5 inches		log. 1.1903317
		2
Diameter of piston squared, 240.25		log. 2.3806634
Constant for circular areas, 0.7854		log. 9.8950899
Pressure on piston per square inch, 35 lbs.		log. 1.5440680
Velocity of piston in feet per minute, 165		log. 2.2174839
Constant for horse power, 33000	ar. co.	log. 5.4814861
Gross power of the lesser cylinder, 33.021 horses		log. 1.5187913

The power of the larger cylinder, as developed by the mean pressure of the steam between the pistons, exerting itself on the circular annulus, or the surplus area of the larger piston, is calculated as below, viz.

Diameter of larger piston, 31 inches
Diameter of lesser piston, 15.5 inches

46.5 sum		log. 1.6674530
15.5 difference		log. 1.1903317
Constant for circular areas, 0.7854		log. 9.8950899
Mean pressure of the steam, 10.127 lbs.		log. 1.0054808
Velocity of the piston, 220 feet		log. 2.3424227
Constant for horse power, 33000	ar. co.	log. 5.4814861
Gross power of larger cylinder, 38.22 horses		log. 1.5822642

Therefore by addition, the total gross power of the engine becomes $33.02+38.22=71.24$ horses.

It may prove to be a matter of interest in this place, to compare the power of the combined cylinder engine which we have just calculated, with that of a single cylinder expansive engine, consuming the same quantity of steam, and occupying the same cylindrical space, admitting the initial force of the steam to be the same in both cases.

In order therefore to make the comparison, it becomes necessary to throw both cylinders into one having the same diameter as the larger; that is, 31 inches, and in order to arrive at a perfectly rigorous inference, it is also necessary to increase the enlarged cylinder, by the space which has been allowed for the steam passages; it is obvious, since the diameter of the cylinder remains the same, that the length will be increased in proportion as the capacity is increased.

Now, we have seen that the absolute capacities of the cylinders, are 5.8966, and 31.4488 cubic feet, and one-half of a cubic foot has been allowed for the steam passages between the cylinders; consequently, the entire capacity of the single cylinder equivalent to the whole, will be $5.8966+31.4488+0.5=37.8454$ cubic feet; but the steam fills a space equal to the capacity of the lesser cylinder, or 5.8966 cubic feet under full pressure, or before the expansion begins; therefore, if the entire capacity of the enlarged cylinder be divided by the capacity of the lesser one, the quotient will be the number of times that the steam dilates itself; hence we have

$$n=37.8454\div 5.8966=6.418, \text{ the relative expansion.}$$

Now, the formula for the mean pressure which is due to the entire effect of the steam, admitting it to enter the cylinder with an initial force of 35 pounds upon the square inch, is

$$\textit{mean pressure}=\frac{35}{6.418}\times(1+\text{hyp. log. } 6.418).$$

But the hyperbolic logarithm of 6.418, is $0.8073997\times 2.3025851$ 1.8591065, and this increased by unity becomes 2.8591, omitting the other figures as superfluous, therefore we have

$$\textit{mean pressure}=\frac{35\times 2.8591}{6.418}=15.592 \text{ pounds per square inch.}$$

It is here evident, that in order to have a just comparison by consuming the same quantity of steam, that whatever number of strokes the engine made in the former arrangement, must also be made in this. Now, the whole capacity of the cylinder in the present instance, is 37.8454 cubic feet, and the capacity of the larger cylinder in the foregoing arrangement, was 31.4488 cubic feet; therefore we get

$$31.4488 : 6 :: 37.8454 : 7.22 \text{ feet, the length of the stroke.}$$

But in the combined cylinder arrangement, the number of double strokes per minute was $18\frac{1}{3}$, for $220\div 12=18\frac{1}{3}$; consequently, it is $7.22\times 18\frac{1}{3}\times 2=264.75$ feet per minute for the velocity of the piston. Therefore, by the rule for computing the power of the engine, we have the following logarithmic operation, viz.

Diameter of the piston, 31 inches . .		log. 1.4913617
		2
Diameter of piston squared, 961 . .		log. 2.9827234
Constant for circular areas, 0.7854 . .		log. 9.8950899
Velocity of the piston, 264.75 feet per minute		log. 2.4228360
Mean pressure of the steam, 15.592 lbs. .		log. 1.1929018
Constant for horse power, 33000 .	ar. co.	log. 5.4814861
Gross power of the engine, 94.414 horses .		log. 1.9750372

Here then the gross power of the engine is equivalent to 94.414 horses, and allowing two-fifths of the whole to be absorbed in overcoming the accumulated resistances, the effective power becomes $94.414 \times 0.6 = 56.65$ horses, the steam being admitted to the piston under the initial pressure of 35 pounds to the square inch.

From what we have here shown, it appears, that with the same cylindrical space, and using the same quantity of steam, the advantage is in favour of the single cylinder engine by no less than 11.99 horse-power, for $56.65 - 44.66 = 11.99$.

In the above calculation however, a very considerable advantage in respect of power, arises from the increased velocity; but even supposing the velocity to be the same as in the combined cylinder engine, viz. 220 per minute, yet the power of the single cylinder engine will still be the greatest, even with a less quantity of steam.

If we substitute the logarithm of 220 for that of 264.75 in the above calculation, all the other logarithms will remain the same as they are, and the gross power of the engine will then be 78.46 horses; and the effective power becomes $78.46 \times 0.6 = 47.08$, being greater than that of the combined cylinder by 2.42 horses, for $47.08 - 44.66 = 2.42$.

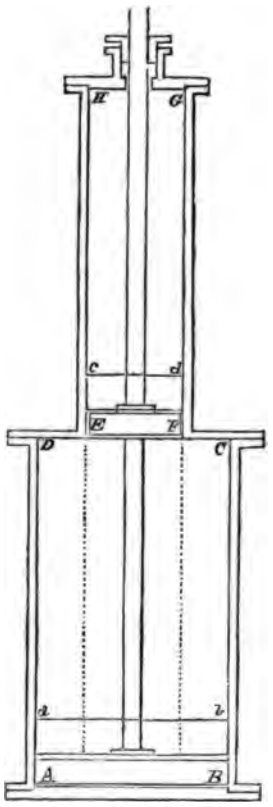
When the piston in the enlarged cylinder is constrained to move with a velocity of 220 feet per minute, the number of double strokes per minute, is 15.235, for we have

$$37.8454 : 18\tfrac{1}{3} :: 31.4488 : 15.235 \text{ very nearly.}$$

Now, in the combined cylinder engine, the number of strokes was $18\frac{1}{3}$, which gives $36\frac{2}{3}$ times the small cylinder full of steam per minute under the pressure of 35 pounds per square inch; whereas with the single cylinder and the same velocity of the piston, the quantity of steam at 35 pounds per square inch, would only be 30.47 times the small cylinder, being a saving of 6.2 cylinders of steam per minute, and a gain of 2.42 horse-power in favour of the single cylinder.

Another form of the combined cylinder expansive engine that claims our attention, is the arrangement introduced by Mr. James Sims, of Redruth, in Cornwall, in which the cylinders are placed the one directly over the other, as shown in the marginal diagram, *fig.* 13; both the pistons being connected together, and worked by the same piston-rod.

Fig. 13.



By this arrangement the stroke of the piston in both the cylinders is constrained to be the same length, the pistons ascending and descending simultaneously to the same extent by reason of the manner in which they are connected; but the diameters of the cylinders themselves differ from each other in such a manner, as to have their capacities in any proportion that is found convenient, or such as the conditions and circumstances of the case may happen to require.

One very striking and important peculiarity of this construction, consists in keeping up a constant vacuum between the pistons by means of a communication with the condenser, and the effect of this is, that both pistons are working against a vacuum at one and the same time, the upper piston being working against the vacuum in the intermediate space, and the lower one against the vacuum in the condenser, so that an extra power is gained, proportional to the difference between the areas of the pistons, or the annulus comprehended between their circumferences when projected on the same plane.

In this mode of construction, the steam is received from the boiler at the top of the lesser cylinder; but when both pistons are at the top of their stroke, there is no steam in the engine, for that which was under the larger piston has just been condensed, by opening a communication with the condenser.

At the instant the condensation is completed, the communication with the boiler is opened, and a fresh supply of steam is admitted above the lesser piston, the downward stroke is then effected by steam under full pressure, acting on the upper side of the lesser piston, against a vacuum formed on the under side of the larger one, and a constant vacuum between the pistons, maintained by means of a communication with the condenser.

When the downward stroke has been completed by full pressure steam acting on the lesser piston, a communication is instantly opened between the top of the lesser cylinder and the bottom of the larger one, under the piston of which, the steam instantly flows from the lesser cylinder, and effects the upward stroke by expansion alone.

At the instant the upward stroke has been completed, the expanded steam is condensed, and a vacuum formed beneath the larger piston, the passage between the cylinders being closed by the same movement, so that steam is again admitted from the boiler to the upper side of the lesser piston, where it acts with full force against a vacuum on the under side of the larger one, thereby causing the cotemporaneous descent of both pistons, without receiving obstruction from any antagonistic influences, with the exception of what may happen to arise from imperfect condensation, which in the present improved state of practical operations, is very trifling. When the pistons have arrived at the bottom of their respective cylinders, as represented in the above figure, the upper cylinder EFGH, is filled with steam at full pressure as received directly from the boiler, while the space ABCD between the pistons is a void, in consequence of a constant communication being kept up between that space and the condenser.

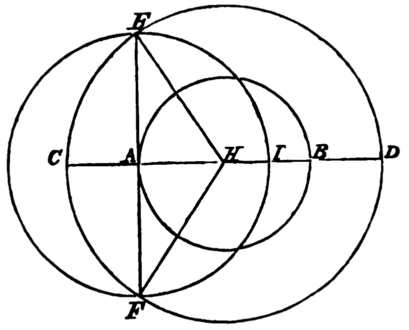
In this state of things, the communication between the boiler and the upper cylinder is cut off, as well as that between the condenser and the bottom of the larger cylinder, while at the same time, the passage between the two cylinders is opened, by which means, the steam is permitted to flow from the lesser cylinder, and expand itself in the greater one, thereby effecting the upward stroke of the engine after the manner already described.

It will here however be seen, that the steam in expanding, exerts an equal pressure on the same quantity of surface in both the cylinders, operating to raise the one upwards, while at the same time it endeavours to press the other downwards, so that the force in opposite directions is lost; consequently, that which raises the piston and works the upward stroke, is the force arising from the expanded steam in the larger cylinder, exerting itself on the difference between the areas of the two pistons, or as we have already stated, on the annulus comprehended between their circumferences when reduced to the same plane.

Now according to the rules of mensuration, this difference of areas is represented by the term 0.7854 × (*greater diameter squared, minus lesser diameter squared;*) but there is an elegant geometrical method of representing the same thing, which, as it may be new to some of our practical readers, we think proper to present it in this place, being desirous of communicating all the useful information that lies in our power, in as far at least, as is consistent with the nature and plan of the present performance, or has a tendency to simplify the subject which we are now discussing, and endeavouring to elucidate.

Let AB, *fig.* 14, be the diameter of the lesser cylinder, and CD the diameter of the greater, the common centre, when referred to the same plane being at H; then will the circles described on AB and CD, comprehend the annulus. Through A the extremity of the lesser diameter AB, and perpendicular thereto, draw the tangent line EF, to meet the circumference of the greater circle both ways in the points at E and F; then is EF, the tangent thus limited, the diameter of a circle which is equal in area to the difference of the two circles, whose diameters are AB and CD respectively, or equal to the annulus comprehended between their circumferences.

Fig. 14.



Upon the tangent line EF, limited in both directions, as a diameter, and with A the point of contact as a centre, describe the circle EIF, which will therefore be equal to the difference between the areas of the greater and lesser pistons, or to the area of the annular space comprehended between their circumferences when projected on the same plane, and this is equal to that portion of the larger piston, on which the expanded steam is unopposed, and on which it exerts its power to effect the upward stroke of the engine; the pressure on the other or central part, being neutralized by an equal pressure on the lesser piston, exerted in an opposite direction.

The truth of this construction is perfectly obvious, for the area of the annular space, or that comprehended between the circumferences of the

the height *ab* by means of the steam expanding underneath it, and pressing on its under side, the lesser piston has been raised to the height *cd*, in consequence of their mutual connection by the same piston-rod.

It will now be manifest, that the steam which is above *cd* in the lesser cylinder, is not of the same density or elastic force, as it was before the piston began to ascend, for a portion of it has passed into the larger cylinder, and there, by occupying a larger space, it has become proportionally rarefied, and in consequence has lost a part of its elastic power.

But the degree of rarefaction is the same in both the cylinders, and also in the passages between them, so that the effect of the steam to raise the pistons, is opposed by an equal pressure endeavouring to depress them; but this antagonistic pressure acts only on the surface of the lesser piston, and for this reason, the force which becomes effective in working the upward stroke, is the mean pressure of the expanding steam, operating on the circular annulus before alluded to, being as there demonstrated, equivalent to the difference between the areas of the two pistons, on which the steam is acting at the same time, but in opposite directions.

Now, in order that the upward and the downward stroke of the engine may be effected by the same motive power, the diameters of the cylinders must be so related to each other, that the mean pressure of the steam under expansion, acting on the circular annulus, shall be equal to the full pressure steam acting on the lesser piston.

Disregarding for the present, the steam passages between the cylinders, it may be instructive to show, in what manner the mean pressure of the expanded steam may be approximated, supposing it to occupy the greater cylinder only, when expanded to its full extent; that is, supposing it to flow immediately from the lesser into the greater cylinder, without any intermediate passage. We have adopted this course in the previous cases, and consequently, for the sake of system, it is incumbent on us to give a similar illustration of the engine we are now considering.

To estimate the power of this engine, it is manifest that we must first of all determine the mean force of the steam as developed by expansion in the larger cylinder, and for this purpose, we must bear in mind, that when expansion takes place in a larger vessel than that in which the steam acts at full pressure, the number of times that the steam dilates itself in the larger vessel, is expressed by the quotient that arises, when the capacity of the larger vessel or cylinder is divided by the capacity of the lesser; or in other words, it is the reciprocal of the fraction which expresses the ratio of the capacities of the two cylinders.

This quotient corresponds to what we have hitherto denominated the index of expansion, and in order to express it in general terms having reference to the dimensions of the cylinders, we propose the following notation.

Let $D = AB$, the diameter of the larger cylinder (see *fig.* 13);
$d = EF$, the diameter of the lesser cylinder, in which the steam acts with full force;
$l = AD$, the length of the larger cylinder;
$l' = EH$, the length of the lesser cylinder;
and $n =$ the index of expansion, or the number of times that the steam is rarefied in filling the larger cylinder, on the supposition that it is wholly expelled from the lesser one.

Then since by the common and well-known rules of mensuration, the capacities of the cylinders are in the compound ratio of their lengths and the squares of their diameters, we shall have, the capacity of the lesser cylinder to that of the greater, as $l'd^2$ to lD^2; consequently, $n = \frac{lD^2}{l'd^2}$; but by the principles of construction, l and l' the lengths of the cylinders are equal to each other, and by the elements of geometry, when the lengths are equal, the capacities are to each other as the squares of the diameters simply, hence we get, as follows for the relative expansion of the steam in the larger cylinder.

$$n = \textit{index of expansion} = \frac{\textit{greater diameter squared}}{\textit{lesser diameter squared}}.$$

If the steam be allowed to expand itself in the lesser cylinder, for a portion of the stroke before it begins to flow into the larger one, where it acts wholly by expansion; then the expression for the value of the index will be different from that which we have given above. It is however unnecessary to consider this case of the question in the present instance, as it is very seldom that the principle of expansion is carried to such an extent as to begin the dilatation in the lesser cylinder.

Supposing therefore, that the steam enters into the larger cylinder under the same initial pressure as it has in the lesser cylinder, which must obviously be very nearly the case at the instant of influx; then the steam acts by expansion throughout the whole length of the stroke in the larger cylinder, and the formula for the mean pressure, is

$$\textit{mean press.} = \frac{\textit{lesser dia. squared} \times \textit{initial press.}}{\textit{greater dia. squared.}} \times \text{hyp. log.} \frac{\textit{greater dia. squared}}{\textit{lesser dia. squared}}.$$

In which equation, the initial pressure is that exerted by the steam throughout the whole length of the stroke in the lesser cylinder, and it is that which it exerts in the greater cylinder also, at the moment of entrance, or just when the expansion begins.

We shall now proceed to show in what manner the mean pressure of the steam is to be approximated, and for this purpose we shall propose the following numerical example.

Example.—Suppose the diameters of the cylinders to be respectively 15.5 and 31 inches; what is the mean pressure per square inch on the greater piston, that on the lesser one as received directly from the boiler being 35 pounds, the steam operating throughout the greater cylinder entirely by expansion?

Since the diameter of the cylinder is just double that of the other, the lengths being equal, it follows from the principles of geometry, that the capacity of the greater cylinder, is just quadruple that of the lesser, the relative capacities may therefore be represented by the numbers 1 and 4, for any similar multiples or sub-multiples of these, will have to each other precisely the same relations as the actual capacities themselves, and in this way the labour of calculation is considerably abbreviated, while the same results are obtained.

Let us now suppose that the pistons, which in *fig.* 13, are represented at the lower limit of their range or stroke, are raised through a tenth part of the length, in consequence of the steam which flows from the lesser cylinder, and expands itself under the piston of the greater; then it is obvious, that since the lengths of the cylinders are equal, each of the pistons will have passed through a tenth part of the length of its respective cylinder, and generated a tenth part of the capacity; and in consequence of this, the steam which previously filled the whole of the lesser cylinder under full pressure, is now contained in nine-tenths of that cylinder together with one-tenth of the larger one, as will readily appear by inspecting the drawing *fig.* 13, where the pistons are supposed to be at the transverse lines *ab* and *cd* respectively.

Under these conditions therefore, the steam is dilated into a larger space, inasmuch as the tenth part of the larger cylinder exceeds a tenth part of completely neutralized by an equal pressure acting in an opposite direction on the upper side of the lesser piston. The above mean pressure is consequently only effective on as much of the larger piston as its area exceeds that of the lesser one, or the area of the annulus comprehended between their circumferences when projected on the same plane, as we have already intimated in another part of the work.

By applying the other approximating rule which we have given for the value of the mean pressure, we find it to be 12.477 pounds per square inch, differing from the aforesaid value by $14.942 - 12.477 = 2.465$ pounds. The operation by this method is as follows.

Sum of the extreme pressures, . .	25.673
Four times the sum of the even pressures,	242.312
Twice the sum of the odd pressures, .	106.340
Sum	=374.325

which being multiplied by one-third of the common interval, that is, $\frac{1}{3}$ of $\frac{1}{10}=\frac{1}{30}$, we obtain $374.325 \times \frac{1}{30} = 12.477$ pounds per square inch very nearly.

If the number of terms in the series; that is, the number of positions of the piston during the stroke were increased, the mean value of the pressure would be more correctly approximated, and by making the number of terms infinite we should thereby obtain the correct mean. But if the steam passages between the upper end of the lesser cylinder and the under end of the larger were taken into account, together with the spaces allowed for the clearance of the pistons at the extremities of the stroke, the foregoing series of results would all be different from what they are in the present instance; for in that case, the constant term would not be 35 as it now is, but something less in consequence of the augmentation of the space in which the steam is originally contained, and that in which it is expanded.

The method of calculation however, would not be more difficult, for the extraneous spaces mentioned above being constant in magnitude, the first term of the series only would be effected thereby, the same constant difference being successively applied for obtaining the first term in the several analogies.

If we apply the finite equation given on a former page, to determine the mean pressure during expansion, on the supposition that the initial pressure of the steam on the larger piston is the same as the constant pressure on the lesser one; then we shall have

Now the quantity of mean pressure per inch exerted on the larger piston, is expressed by the term $\frac{d^2P}{D_2}\left\{\text{hyp. log. }\frac{D_2}{d^2}\right\}$; consequently, the momentum or mechanical effect of the steam in working the upward stroke, is as follows, viz.

$$\textit{upward momentum}=\frac{0.7854(D^2-d^2)d^2lP}{D^2}\left\{\text{hyp. log.}\frac{D^2}{d^2}\right\}$$

This is the momentum of the larger cylinder, or that by which the upward stroke is effected, and when given in a specific form, it becomes

$$\textit{momentum}=\left\{0.7854(\textit{greater dia.}^2-\textit{lesser dia.}^2)\times\textit{lesser dia.}^2\times\textit{length}\times\textit{initial press.}\times\text{hyp. log.}\frac{\textit{greater dia.}^2}{\textit{lesser dia.}^2}\right\}\div\textit{greater dia.}^2$$

If to the momentum of the larger cylinder as thus obtained, the momentum of the lesser cylinder be added, the entire momentum of the engine will be as follows, viz.

$$\textit{total momentum}=0.7854d^2lP\left\{1+\left(\frac{D^2-d^2}{D^2}\right]\times\text{hyp. log. }\frac{D^2}{d^2}\right\}.$$

This gives the total momentum of the engine for the entire stroke, on the supposition that the diameters of the cylinders, the length of the stroke, and the initial pressure of the steam are all given, without having regard to the equality of the mechanical effect in the downward and the upward strokes.

But when it is required that the effect shall be the same in both strokes, the diameter of one of the pistons must be known *a' priori*, and from it the other must be computed in such a manner, that the steam acting by expansion on the annulus between their circumferences shall produce the same effect as the full pressure steam on the lesser piston. In order therefore that the effects shall be equal on both pistons, we must compare the momenta of both pistons as above computed, according to the forms in which we have represented them. Thus we have

$$d^2lP=\frac{(D^2-d^2)\,d^2lP}{D^2}\times\text{hyp. log. }\frac{D^2}{d^2}.$$

Now, in this equation, d^2lP enters on both sides, consequently, by eliminating these quantities from both, we obtain

$$\frac{(D^2-d^2)}{D^2}\,\text{hyp. log. }\frac{D^2}{d^2}=1,$$

which finally becomes as follows,

$$(D^2-d^2)\times\text{hyp. log. }\frac{D^2}{d^2}=D^2.$$

In which equation, either D or d may be considered as the unknown quantity, according as the one or the other is given. If the same relation be expressed in terms of the common logarithms, it becomes,

$$(D^2-d^2)\times\log.\frac{D^2}{d^2}=0.4342945\,D^2\,;$$

and the same may be otherwise expressed as follows, viz.

$$\log.\,D-\log d=0.2171472\,\frac{(\;D^2\;)}{D^2-d^2}.$$

Now, supposing the value of the lesser diameter to be given, that of the greater can be found from the equation in the form which it has now assumed; or if the value of the greater diameter is given, that of the lesser one can be found; it must however be observed, that the arrangement of the equation is such as can only be reduced by approximation: but when a table of logarithms is at hand, the solution is very easily effected, and since in practical cases, the result can never be required to any great degree of nicety, the necessity of having recourse to this mode of solution is of very little consequence, and must in every case be adopted, since a direct solution from the formula cannot in this instance be obtained; but one trial or at most two, will be sufficient to give a result very near the truth.

Supposing therefore that the diameter of the lesser piston is given, and that it is required to determine that of the larger one, so that the momentums of both cylinders may be the same; or which is the same thing, that the lesser piston may descend under the same pressure as the larger one ascends.

By referring to the foregoing equation, it will at once be perceived that it is wholly independent of the initial pressure of the steam, or any other quantity except the diameters of the pistons and known numbers, the reduction therefore is very easily effected in the following manner.

Let the diameter of the lesser piston be 15.5 inches, what must be the diameter of the greater one, in order that the mechanical effect may be the same on both, the one working under full pressure, and the other by expansion?

Now, a single trial will show us, that the required diameter is greater than 30 and less than 31 inches; for if we take 30 inches, the terms on the left hand side of the equation give

$$\log.\,D-\log.\,d=\log.\,30-\log.\,15.5=1.4771213-1.1903317=0.2867896,$$

and on the same supposition, the compound term on the right hand side gives

$$\frac{0.2171472 \times 30^2}{(30+15.5) \times (30-15.5)} = 0.2962220,$$

which differs from the above result by $296222 - 2867896 = 94324$ in excess.

Again, if we take 31 inches, the terms on the left hand side of the equation give

$\log. \text{D} - \log. d = \log. 31 - \log. 15.5 = 1.4913617 - 1.1903317 = 0.3010300$,

and on the same supposition, the term on the right hand side gives as below.

$$\frac{0.2171472 \times 31^2}{(31+15.5) \times (31-15.5)} = 0.2895296,$$

being less than the above difference by 115004, for $3010300 - 2895296 = 115004$; therefore by the rule of position, we have the following process for the first approximation.

First supposition, $30 \times 115004 = 3450120$ product by the second error.
Second supposition, $31 \times 94324 = 2924044$ product by the first error.

Sum of the errors $= 209328$ 6374164, sum of the products,

Therefore, if the sum of the products be divided by the sum of the errors, we obtain for the first approximate value of the greater diameter as underneath, viz,

greater diameter $= \dfrac{6374164}{209328} = 30.45$ inches; being very nearly double the lesser diameter.

If we make trial with this result as one of the supposed numbers, the terms on the left hand side of the equation will give

$\log. \text{D} - \log. d = \log. 30.45 - \log. 15.5 = 1.4835873 - 1.1903317 = 0.2932556$,

and by substituting 30.45 for D, the term on the right hand side of the equation produces the following result, viz.

$$0.2171472 \frac{\text{D}^2}{(\text{D}^2 - d^2)} = 0.217472\left(\frac{30.45^2}{30.45^2 - 15.5^2}\right) = 0.2930907,$$

which differs from the above remainder by 0.0001649, for we have $0.2932556 - 0.2930907 = 0.0001649$; therefore, by the rule of position, we have

First supposition, . $30 \times 1649 = 49470.0$, product by the second error.
Second supposition, $30.45 \times 94324 = 2872165.8$, product by the first error.

Sum of the errors $= 95973$ $\quad 2921635.8$, sum of the products.

Therefore, if the sum of the products be divided by the sum of the errors, the second approximate value of the required diameter becomes

$$\frac{2921635.8}{95973} = 30.441 \text{ inches nearly.}$$

And by again repeating the process by the last found number, a more correct value will still be obtained; but by substituting the present result in the general equation, it will be found very nearly to succeed; so that 30.441 inches may be taken as the diameter required.

Now, it may here be remarked, that whatever is the ratio of the capacities of the cylinders in this case, will hold in every other case under the same conditions. In order therefore to determine the ratio, we must divide the square of the one diameter by the square of the other; hence it is

$$\frac{15.5^2}{30.441^2} = \frac{1}{3.857};$$ or in round numbers, as 1000 to 3857.

Consequently, when the diameter of the lesser cylinder is given, and it is required to find that of the greater, the following is the rule :—

> RULE.—*Multiply the square of the lesser diameter, by the constant number,* 3857, *and divide the product by* 1000; *then the square root of the quotient will be the diameter of the larger cylinder.*

And when the larger diameter is given, to find the lesser one under the same conditions, the rule for effecting it is as follows :—

> RULE.—*Multiply the square of the given diameter by* 1000, *and divide the product by* 3857; *then the square root of the quotient will be the lesser diameter required.*

Example.—Suppose the diameter of the lesser cylinder to be 28 inches; what must be the diameter of the larger cylinder, so that the downward and the upward stroke may be effected by the same mean force ?

Operating by the first of the foregoing rules, we have $1000 : 784 :: 3857 : 3023.888$, and extracting the square root of this, it gives,

$$\textit{greater diameter} = \sqrt{3023.888} = 54.99 \text{ inches.}$$

Again, suppose the greater diameter to be given equal to 55 inches; what must be the lesser diameter, so that the upward and the downward stroke may be effected by the same mean force?

Operating by the second of the foregoing rules, we have, 3857 : 3025 :: 1000 : 784.2883, and extracting the square root of this, we get,

$$\textit{lesser diameter} = \sqrt{784.2883} = 28.005 \text{ inches.}$$

Since we have found the general relation of the two cylinders, to be as the numbers 1000 and 3857; it follows, that the relative expansion of the steam in the larger cylinder, will be 3.857 times; and consequently, when the initial pressure is 35 pounds per square inch, the mean pressure by expansion in the larger cylinder will be as below, viz.

$$\textit{mean pressure} = \frac{35}{3.857} \times \text{hyp. log. } 3.857 = 12.249 \text{ pounds per square inch.}$$

Consequently, if relative proportion between the capacities of the cylinders be as we have found it above, we should have the following equation; that is,

$$\textit{lesser dia. squared} \times \textit{initial press.} = (\textit{greater dia.}^2 - \textit{lesser dia.}^2) \times \textit{mean press.}$$

And accordingly, by substituting the numerical values of the several quantities for their respective symbols or representatives, we get

$$28.005^2 \times 35 = (55^2 - 28.005^2) \times 12.249.$$

Now, when the calculation is carried out, we find the product on the left hand side of the equation to be 27449.80, and that on the right hand side is 27447.56, as near a coincidence as can be expected under the circumstances; differing by only $2\frac{1}{4}$ pounds on the whole effective area.

An engine constructed on this principle, whose cylinders have to each other the ratio which we have specified above, will perform the downward and the upward strokes with the same quantity of absolute power, and with one small cylinderful of steam only, the downward stroke being performed with steam at full pressure, and the upward one by the same steam expanded in the larger cylinder.

The economising of steam is indeed the chief advantage which the present mode of construction offers, the steam from the boiler being required to fill the lesser cylinder only, in which it acts with full pressure against a vacuum on the under side of the larger piston; it is then permitted to flow into the larger cylinder, where it developes on equal mean force by acting expansively on a larger surface under a diminished intensity of pressure.

A farther advantage may however be attained, by working the steam expansively in the lesser cylinder also; but in that case it must be admitted from the boiler under a much higher degree of elasticity; if not, it may be so much attenuated when expanded in the larger cylinder, as to be incompetent of carrying the pistons to the top of the stroke.

The mode of calculation, even under a double expansion, is sinilar to that which we have already exemplified, for when the steam from the boiler is cut off at any proposed part of the stroke, we have only to take the mean force of the steam in the lesser cylinder to calculate the power of the engine, and its terminal force will then become the initial pressure for calculating the mean in the larger cylinder, as developed by the second process of expansion. The calculation in every other respect is the same as we have previously illustrated; but it will probably be better to represent this new condition by an expression adapted to the purpose, and by resolving a particular numerical example, in which the effect of the double expansion will become manifest.

The equation exhibiting the relation of the diameters under these circumstances, is as follows, viz.

$$\log. \mathrm{D} - \log. d = \frac{\mathrm{D}^2(\log. n + 0.4342945)}{2\,(\mathrm{D}^2 - d^2)}$$

This formula is only different from that which we employed in the previous case, when expanding in the large cylinder only, inasmuch as it involves the term log. n, which necessarily varies the co-efficient of the term D^2; but in every other respect the mode of operation is the same as that which we have already exemplified, when there was no expansion in the lesser cylinder, but in order that everything may be rendered as plain as possible, we think proper to propose and resolve the following numerical example.

Example.—Suppose the steam from the boiler, to enter the lesser cylinder under a pressure of 35 pounds to the square inch, and to be intercepted when the piston has traversed a fourth part of the stroke; what must be the diameter of the larger cylinder, so that the mean effect may be the same on both pistons, the lesser diameter being 28 inches?

Since the steam is cut off at one-fourth of the stroke, the value of the symbol n, or the number of times that the steam expands itself n the lesser cylinder, is 4; and consequently, the terminal pressure in that cylinder is 8.75 pounds per square inch, which is also the pressure under which the the expansion in the larger cylinder begins; but since the first expansion takes place in the same cylinder that the steam enters from the boiler under full pressure, its mean pressure in that cylinder, is represented as below, viz.

$$\textit{mean pressure} = 875(1 + \text{hyp. log. } 4) = 20.88 \text{ pounds.}$$

The mean power of the engine is therefore represented by 28×20.88, whereas in the former case, where the steam acted with full power throughout the lesser cylinder, it was represented by 28×35; hence it appears, that with one-fourth the quantity of steam from the boiler, we have 40 per cent. less power, but consume 75 per cent. less steam. It would however be easy to maintain the same power with one-fourth the quantity of steam, by raising it to a sufficient degree of intensity in the boiler; but in this case, it would require an additional quantity of fuel to maintain the necessary temperature.

Reverting to our example in which the value of the symbol n is 4, we shall find that the relation between the diameters of the cylinders is altered in consequence of the first expansion, or that which takes place in the lesser cylinder; for the common logarithm of 4 is 0.6020600, and this added to constant 0.4342945 and divided by 2, gives 0.5181772, so that the theorem from which the diameter is to be found, becomes

$$\text{log. } D - \text{log. } d = 0.5181772\, D^2 \div (D^2 - d^2).$$

and from this theorem the value of D is to be found by trial and error, as in the preceding case of non-expansion in the lesser cylinder.

By a very simple process we find the value of D to lie between 101 and 102 inches, for with 101 substituted according to the formula, we find the error in defect to be 0.0010352; and with 102, the error is 0.0041539 in excess; consequently, by applying the rule of position, the diameter of the larger cylinder comes out 101.8 inches.

If we make trial with the number 101.8, it will be found to succeed very nearly, but in this instance the expansion is very far extended, and in order to render the steam effective throughout the ascending stroke, it must be generated under a very high degree of pressure in the boiler.

In order to ascertain how often the steam dilates itself in the larger cylinder, it becomes necessary to divide the square of the larger diameter by the square of the lesser, and the product will be the relative expansion sought; thus we have

$$101.8^2 \div 28^2 = 10363.24 \div 784 = 13.218.$$

Now, if we suppose the initial pressure of the steam as it enters the lesser cylinder to be 35 pounds upon the square inch, and to be cut off at one-fourth of the stroke: then the terminal pressure in the lesser cylinder will be 8.75 pounds per square inch, and that is also the pressure under which the steam enters the larger cylinder, abstracting for the effect of the steam passages between the cylinders, or from the upper side of the lesser piston to the under side of the greater.

Since the steam expands itself 13.218 times in the larger cylinder, the terminal pressure will be very greatly reduced, but as it operates upon a very large surface, its power to move the engine will still be very considerable; but in order to insure a sufficient motive force throughout the operation, it becomes necessary to raise the steam to a much higher degree of pressure in the boiler, a condition which is attended with some additional expense.

To prove that both cylinders develope the same mean power, according to the dimensions above determined, we must compute the mean pressure of the steam in both, corresponding to the cut-off in the lesser cylinder, and the rate of expansion in the greater. Now, by taking the cut-off in the lesser cylinder at one-fourth the length of the stroke, with a pressure from the boiler of 35 pounds per square inch: then will the terminal pressure in the same cylinder be 8.75 pounds, and the mean pressure 20.88 as we have already found it to be.

Since the terminal pressure in the lesser cylinder is the same as the initial pressure in the greater, and because the capacities are to each other as 1 to 13.218: it follows that the mean pressure in the larger cylinder is 1.71 pounds per square inch: for we have

$$\text{mean pressure} = \frac{8.75}{13.218} \times \text{hyp. log. } 13.218 = 1.71 \text{ pounds nearly.}$$

Therefore, by the conditions of the problem, $28^2 \times 20.88$ ought to be the same as $101.8^2 \times 1.71$; and by performing the process we find the one product to be 16369.92, and the other 17721.3 pounds.

COMPARISON OF THE ENGINES.

Having fully and separately considered the three different kinds of engines mentioned in our premises, as working with steam on the expansive principle, we now propose briefly to recapitulate our former labours, and to make a comparison of the advantages or disadvantages to be obtained by the adoption of the several modes of construction which have been devised for the purpose of working steam expansively.

The first kind of engine that claims our attention according to the plan of arragement which we have adopted, is the Single Cylinder Expansive Engine, or that in which the steam is expanded in the same cylinder where it also acts under full pressure, the supply from the boiler being intercepted when the piston has traversed a determinate portion of its stroke.

In this kind of engine, the advantages to be derived from working the steam expansively are very considerable, especially when the steam is generated under a very high degree of intensity; and the object of the present enquiry is to point out under what conditions and circumstances the greatest possible advantages are to be obtained by the adoption of an engine of this description.

In all calculations respecting the steam engine, whatever may be the nature of its construction, that of determining its power is the most important; this is very easily accomplished in the case of an engine, where the steam acts at full pressure throughout the stroke; but where the expansive principle is brought into operation, other considerations must have place. This is obvious, for a greater or lesser degree of expansion in the cylinder may be required, and in order to obtain the exact degree proposed, the steam must be admitted into the cylinder under a specified pressure, and a particular point of cut-off must also be assigned, in order that the particular object may be attained. Now, our previous investigations enable us to arrive at the knowledge of these desiderata, and it is to this end, that our attention in the present instance is more especially directed.

Supposing that in any case, it is required that the steam from the boiler shall be admitted into the cylinder under a proposed degree of intensity or pressure; it is first of all requisite to know, under what pressure it must be generated in the boiler, and at what temperature it must be maintained to produce that pressure. Now, experiment and attentive observation on select examples, combined with theoretical deductions therefrom, have taught us how to approximate to the amount of loss in the force of the steam in passing from the boiler into the cylinder, arising from various causes; and in a well-constructed condensing engine, working expansively, this amount of loss is reckoned equivalent to two-fifths of the whole power under which the steam is generated, leaving three-fifths thereof as being effective in moving the engine, with its attendant trains of machinery.

The introduction of the foregoing particulars along with our previous investigations, would however, render the formulæ for computing the power of the different sorts of engines too complicated for practical application, at least, in a combined form; and for this reason, we have thought proper to omit the expression which involves these several particulars; but in order that nothing may be omitted that has a tendency to simplify and elucidate the subject, we beg leave especially to call the attention of our readers to what immediately follows.

We have stated above, that after deducting the loss of force in the steam, by cooling in passing along the pipes—in flowing through the valves and other passages—by leakage, and other causes of loss—three-fifths of it becomes effective in working the engine; it therefere follows, that when the pressure under which the steam enters the cylinder is given, that under which it must be generated in the boiler can readily be ascertained, for we have only to take five-thirds of the pressure on entering the cylinder, and the result will be the pressure in the boiler; and when the pressure in the boiler has thus become known, the temperature of generation can readily be determined, either by direct computation, or by reference to tables which have previously been computed for this and kindred subjects.

These particulars being premised, we propose the following case, as the basis of our reasoning on what respects the single cylinder expansive engine, and which also by a little extension, will apply to the consideration of the combined-cylinder engine, according to Wolf's and Sims's arrangement.

General example.—In a single cylinder expansive engine working by condensation, the diameter of the cylinder is 42 inches; the length of stroke 11 feet; the number of double strokes 22; the cut-off at one-fourth of the stroke; and the initial pressure of the steam, or that under which it enters the cylinder, 50 pounds per square inch; what is the momentum or mechanical effect of the engine, the resistance from friction, imperfect condensation, and other causes being disregarded; and under what pressure and temperature must the steam be generated in the boiler, in order that its pressure on entering the cylinder shall be 50 pounds per square inch?

Now, the general formula for computing the power of the single cylinder engine under these conditions, is

$$\textit{power in pounds} = 0.7854 \times \textit{diam.}^2 \times \textit{velocity} \times \textit{mean pressure}.$$

But the mean pressure which is due to the entire effect of the steam in the cylinder is expressed by the term

$$\frac{2.3025851 \times \textit{initial pressure} \times (\log.\ n + 0.4342945)\,;}{n}$$

and substituting this term for the mean pressure in the above expression for the momentum or power of the engine, we get as follows.

$$\textit{power in pounds} = \frac{1808446\ \textit{diam.}^2 \times \textit{vel.} \times \textit{initial pres.} \times (\log.\ n + 0.4342945)}{n}.$$

In consequence of the manner in which the several members of the preceding formula are combined with one another, it is very well adapted for logarithmic computation, we therefore prefer drawing up the following solution in that way, as it requires much less mental labour than would be required by the common arithmetical process.

Diameter of the piston, 42 inches	log. 1.6232493
	2
Diameter of the piston squared, 1764	log. 3.2464986
Velocity of the piston, 22 × 22 = 484 feet	log. 2.6848454
Initial pressure of the steam, 50 pounds	log. 1.6989700
Log. $n + 0.4342945 = 0.6020600 + 0.4342945 = 1.0363545$	log. 0.0155084
Constant coefficient, 1.808446	log. 0.2573056
Index of expansion, 4	ar. co. log. 9.3979400
The natural number or momentum, 20001750 pounds	log. 7.3010680

The number just obtained, is the number of pounds raised one foot per minute, which are equivalent to the momentum or power of the engine, and represents its competency to produce mechanical effect; and if we divide that number by 33000, the quotient will give the horse-power to which the engine is equivalent, omitting the effect of friction, imperfect condensation, and other resistances to which the working of a steam-engine is liable. It will however, answer our present purpose more effectually, to retain the expression of the power in pounds, as affording a readier means of comparison with other quantities of the same kind expressed in the same measures.

Since the initial pressure of the steam as it enters the cylinder is 50 pounds upon a square inch of the piston, its pressure when referred back to the boiler, must be $\frac{50\times5}{3}=83\frac{1}{3}$ pounds; and since one pound at a rough estimation is equivalent to two inches of the mercurial column, the entire pressure in the boiler when estimated in inches of mercury, is $83\frac{1}{3}\times2=166\frac{2}{3}$; which corresponds to a temperature of about 315 degrees of Fahrenheit's thermometer, and this temperature must be constantly maintained, in order that the steam may enter the cylinder under a pressure of 50 pounds per square inch.

To pursue the subject a little farther, let the diameter of the cylinder, its velocity, and the initial pressure of the steam, remain as in the foregoing example, and let the point of cut-off be at one-eighth of the length of the stroke; what then will be the momentum or mechanical effect of the engine under these conditions?

Since everything remains as in the preceding case, with the exception of the point of cut-off, it is perfectly manifest, that no other change can take place in the power of the engine than what arises from using a less quantity of steam, and consequently, no alteration takes place in the foregoing computation, beyond the two terms into which the index of expansion enters: the mode of operation is therefore as follows.

Diameter of the piston, 42 inches . .		log. 1.6232493
		2
Diameter of the piston squared, 1764 .		log. 3.2464986
Velocity of the piston, $22\times22=484$ feet .		log. 2.6848454
Initial pressure of the steam, 50 pounds .		log. 1.6989700
Log. $n+0.4342945=0.9030900+0.4342945$ $=1.3373845$		log. 0.1262563
Constant eoefficient, 1.808446 . . .		log. 0.2573056
Index of expansion, 8 . . .	ar. co.	log. 9.0969100
Natural number or momentum, 12905826 pounds		log. 7.1107859

Here then it appears, that with one-half the quantity of steam admitted into the cylinder, that we had in the preceding case, we obtain a momentum of 12905826 pounds; from which we infer, that the saving of steam by an earlier cut-off, or, in other words, by a higher degree of expansion, is much greater in proportion than the corresponding diminution of mean power; for if they were in the same proportion, the last computed momentum would have been only 10000875 pounds, instead of 12905826, as we have found it to be, thereby giving a difference of 2904951 pounds in the power of the engine, which is equivalent to 88 horses.

From this comparison of the results obtained from the two different points of cut-off, all other things remaining the same, the great advantage to be derived by carrying out the expansion is obvious; but at the same time it must be observed, that there are limits beyond which it must not be extended; for if it be carried too far, the momentum of the power may become less than the momentum of the resistance, and in such a case, no motion would be produced. In fact, the extreme point to which the expansion of the steam may be advantageously carried out, can only be assigned when the total resistance to be overcome is correctly known; for the mean pressure due to the entire effect of the steam, corresponding to any extent of cut-off, must always be of sufficient intensity to give the motive power a preponderance over the resistance, from whatever sources it may arise.

To take another view of the subject; let us suppose the initial pressure of the steam as it enters the cylinder to be so much increased in the latter case above illustrated, that the momentum shall be precisely the same with the latter cut-off as it is with the greater, all other things remaining as before.

Here then it is manifest, that in order to make a comparison in this instance, we must first determine what the initial pressure ought to be; and since we have not hitherto shown how this determination is to be effected, in terms of the diameter and velocity of the piston, we shall so transform the foregoing equation, as to represent the value of the initial pressure required, in terms of the appropriate quantity; thus we obtain

$$\textit{initial pressure} = n \times \textit{power in pounds} \div 1.808446 \times \textit{diam.}^2 \times \textit{velocity} \\ \times (\log. n + 0.4342945)$$

Now nothing can be more easy than to assign the initial pressure of the steam from this equation, because the several constituent quantities are so combined with one another, that the entire process can be effected by logarithms; and since it is always preferable to employ this mode of computation when practicable, we shall adopt it in the present instance. The operation is as follows.

Computed momentum for the cut-off at $\frac{1}{4}$th, 20001750 pounds	log.	7.3010680
Index of expansion, 8	log.	0.9030900
Diameter of piston squared, 1764	ar. co. log.	6.7535014
Velocity of piston, 484 feet	ar. co. log.	7.3151546
Log. $n+0.4342945=0.9030900+0.4342945=1.3373845$	ar. co. log.	9.8737437
Constant coefficient, 1.808446	ar. co. log.	9.7426944
Natural number, or initial pressure, 77.491 pounds	log.	1.8892521

It therefore appears, that steam entering the cylinder under the initial pressure of $77\frac{1}{2}$ pounds to the square inch, will develope the same amount of mean power with a cut-off at $\frac{1}{8}$th, that steam of the initial pressure of 50 pounds to the square inch will develop with a cut-off at $\frac{1}{4}$th. Now this result, as well as that which we have previously obtained, says much in favour of the expansive principle; for it is very obvious, that the extra expense incurred by the consumption of fuel, in raising the steam from $88\frac{2}{3}$ to 129 lbs. cannot be equal to the saving effected by employing only one half the quantity of steam at the higher pressure.

In order to prove that the power developed by steam at $77\frac{1}{2}$ pounds pressure, and one-eighth cut-off, is the same as that developed with steam at 50 pounds pressure and one-fourth cut-off, we have only to show, that

$$\frac{50}{4}\times(\text{hyp. log.}\,4+1)=\frac{77.491}{8}\times(\text{hyp. log.}\,8+1).$$

Therefore, by reducing these two expressions, we find the value in the one case to be $\frac{50}{4}\times(1.3862944+1)=12.5\times2.3862944=29.82868$; and in the other, it is $\frac{77.491}{8}\times(2.0794415+1)=9.6864\times3.0794415=29.82868$, an exact coincidence, showing that our deductions are strictly and rigorously correct.

By pursuing a reverse course to the above, it may in like manner be shown, that with steam of a lower initial pressure than 50 pounds to the square inch, it would require a much longer cut-off than one-fourth of the stroke to develope the same quantity of mean power; so that the waste of steam at the lower pressure, would be much greater in proportion than the saving in the fuel required to produce it; which affords another proof of the advantage to be gained by using steam of a high elastic power.

Let us for example, take the cut-off at one-half the length of the stroke, which will require twice the quantity of steam that was employed in the first case when the initial pressure was 50 pounds per square inch, and the cut-off at one-fourth the length of the stroke, only the steam in this latter case will be of less density, and consequently of less elastic force. In carrying out the calculation for the initial pressure with this change of data, we have only to substitute the proper index of expansion in the formula adapted to the purpose, and with this substitution, the operation will in every respect be the same as before. It is as follows.

Computed momentum, 20001750 pounds .	log.	7.3010680
Index of expansion, 2	log.	0.3010300
Diameter of piston squared, 1764 .	ar. co. log.	6.7535014
Velocity of piston, 484 feet . .	ar. co. log.	7.3151546
Log. $n+0.4342945=0.3010300+0.4342945=0.7353245$	ar. co. log.	0.1335209
Constant coefficient, 1.808446 .	ar. co. log.	9.7426944
Natural number, or initial pressure, 35.2346 pounds	log.	1.5469693

It therefore appears, that a certain quantity of steam at 50 pounds initial pressure will produce the same mechanical effect as a double quantity at 35.2346 pounds ; and in order to prove that this is the case, we have only to show, that the following equation obtains, viz.

$$\tfrac{50}{4}\times(\text{hyp. log. }4+1)=\tfrac{35.2346}{2}\times(\text{hyp. log. }2+1).$$

And by reducing these two expressions, we find the value of that on the left hand side to be 29.82868 ; and by exactly a similar process, the value of the right-hand comes out precisely the same, which constitutes a complete proof of the truth of our reasoning on this particular point.

It may be proper here to remark, that the initial pressure in both the preceding cases, can be more easily and expeditiously determined from the expression immediately above, by substituting the letter P for the initial pressure determined by the foregoing operation ; thus we have

$$\tfrac{\mathrm{P}}{2}\times(\text{hyp. log. }2+1)=\tfrac{50}{4}\times(\text{hyp. log. }4+1).$$

In which expression P is the thing required; therefore, if we reduce the equation in reference to P, we obtain

$$\textit{mean pressure}=\mathrm{P}=25\times\frac{(\text{hyp. log. }4+1)}{(\text{hyp. log. }2+1)}=35.2346,$$

the same as above.

Now, in this case the waste of steam is much greater in proportion than the extra expense of fuel required in raising the pressure from 35.2346 to 50 pounds, and for this reason, the greatest advantage would be obtained by using steam of the higher intensity or pressure.

We need not pursue the discussion of the single cylinder engine any farther, for every variety of example would tend to show the advantage of working with high-pressure steam.

In reverting to the second class of engines according to our arrangement, which has a reference to Mr. Wolf's construction, we are strongly of opinion that many of our mechanical engineers labour under a misconception as regards the advantages to be derived from it, and in order to remove this misconception as far as lies in our power, we intend to be somewhat particular in its discussion.

This kind of engine, as we have elsewhere stated, consists of two cylinders of different capacities, placed side by side in the direction of the beam, and connected together in such a manner, that the steam after acting in the lesser cylinder under full pressure, is permitted to flow into the larger cylinder, and there to operate a second time by expansion; and having operated a second time in this manner, is allowed to pass into a separate vessel, where it is finally condensed, and a vacuum formed in the same manner as in the single cylinder engine which we have just been discussing.

Now the idea comprehended under this arrangement is very good in itself, and if it could be practically carried out, would be attended with very considerable advantages; but in consequence of the peculiar arrangement of the parts, the idea involved in the scheme can never be actually realized; for it is supposed that the steam which enters the lesser cylinder directly from the boiler, is acting throughout the stroke with full pressure on the lesser piston, and thereby producing its full effect in giving motion to the engine; while at the same time, the expanded steam is acting in the larger cylinder against a vacuum formed by condensation on the opposite side of the piston; and it is moreover considered, that as both pistons move simultaneously in the same direction, the force exerted by the expanded steam on the larger piston, is wholly additional to the force exerted by the steam at full pressure on the lesser piston.

This however, is not the case, and it is very easy to explain why such a condition does not obtain; for although it is true that the steam from the boiler is pressing with its full force on every square inch of the lesser piston during the stroke, yet it does not produce the same effect in moving the piston, that it would do if it were acting against a vacuum on the other side. It is, in fact, a variable effect, being continually counteracted by the steam between the pistons according to the degree of expansion; varying continually from the instant the steam enters the cylinder where it is least, until the piston has completed its range, and at that point only, does the steam operate with its full effect in moving the piston, or rather in endeavouring to move it, for the piston is then on the point of return, being again subjected to the pressure of the steam from the boiler in an opposite direction.

We have already described the nature of this variable force on the lesser piston in the former part of this work, where we have also illustrated the mode of its operation and effect by means of numerical examples; we need not therefore in this place repeat the operations; we may however just observe in passing, that if by some other means of connexion, or arrangement of the steam passages between the cylinders, the steam could be wholly and instantaneously withdrawn from the lesser cylinder, the instant the piston has completed its stroke, so that the steam from the boiler may act against a vacuum, at the same time that the steam which is expanding in the larger cylinder is acting against a vacuum also; then, all the advantages contemplated by this mode of construction would be completely realized; but as this is a point which is practically unattainable, we may safely infer, that the advantages to be derived from this kind of engine, are more imaginary than real, it being in effect nothing more than the single cylinder engine of equal cylindrical capacity, when the steam is cut off before the piston has completed its stroke, the same quantity of steam being employed in both cases; and when we consider the complicated arrangement, and the consequent additional expense that must be incurred by this mode of construction, we are justified in recording an opinion decidedly unfavourable to the adoption of this species of engine.

In the combined cylinder engine according to Mr. Sims's construction, the objections which we have advanced in the case of Wolf's do not obtain; for since the cylinders are placed the one directly over the other, and both fixed upon the same piston-rod, they must obviously both ascend and descend to the same extent in their respective cylinders; and since the steam from the boiler is only admitted into the lesser cylinder, while a vacuum is formed in the larger by condensation in the usual way, it is evident that this steam operates with full force on the lesser piston throughout the whole of its descent; and for the same reason, when it is permitted to flow into the larger cylinder and there dilate itself, it operates with mean force of expansion in producing the ascent. This force however, is not exerted on the whole surface of the larger piston, but only on that part of it which is in excess of the lesser one, or the annulus comprehended between their circumferences when supposed to be projected on the same plane; the other part being counteracted by the steam as expanded in the upper or lesser cylinder; for it is manifest, that whatever may be the force of the expanded steam on a square inch of the under side of the larger piston, it is of the same force on an inch of the upper side of the lesser one, and therefore operates to counteract the ascent.

In Wolf's engine as we have already seen, the steam is acting on both pistons at the same time; that is, on the lesser at full pressure as it is received from the boiler, and on the larger by expansion; consequently, there are two cylinders full of steam constantly in operation after the engine has been started, and this requires the lesser cylinder to be twice filled with steam at full pressure, to work the ascending and the descending stroke, or to carry the pistons from the bottom to the top, and from the top to the bottom of their respective cylinders; so that in respect to the economy of steam, this kind of engine possesses no advantage whatever, over the double-acting non-expansive engine commonly in use.

In Mr. Sims's arrangement on the other hand, the steam is acting only on one piston at a time; that is, in the upper cylinder it acts at full pressure against a vacuum, and in the larger cylinder it acts by expansion, also against a vacuum; consequently, one cylinderful of steam from the boiler completes the stroke, both descending and ascending; so that an engine of this construction, consumes only one-half the quantity of steam that is consumed by Wolf's, while at the same time it developes a greater and more uniform power, all other conditions and circumstances being equal.

Now, this is an immense advantage possessed by this arrangement; and when we compare the simplicity of its construction with the complicated arrangement of Wolf's, its superior claims to public patronage and support will at once be admitted.

With respect to the advantages to be derived from the use of high or low pressure steam, they are precisely the same here as in the case of the single cylinder expansive engine, and for this reason, whatever we have shown to hold with respect to that, may be applied here with equal propriety and certainty.

Another arrangement of Wolf's principle has lately been patented by Mr. Macnaught of Glasgow, where he places the small cylinder on the other side of the centre of motion; but unless in allowing the principle to be applied to old engines and others already constructed, we can see no advantage to be derived from this arrangement, otherwise than by increasing the the stroke of the lesser piston by placing it nearer to the extremity of the beam when that can be done. Indeed, we consider that it will rather be attended with some disadvantages, for the valves must be worked by a separate apparatus, and the arrangement of the steam passages will also require to be different, and probably more complex.

SECTION VIII.

SIMS'S WHEEL ENGINE.

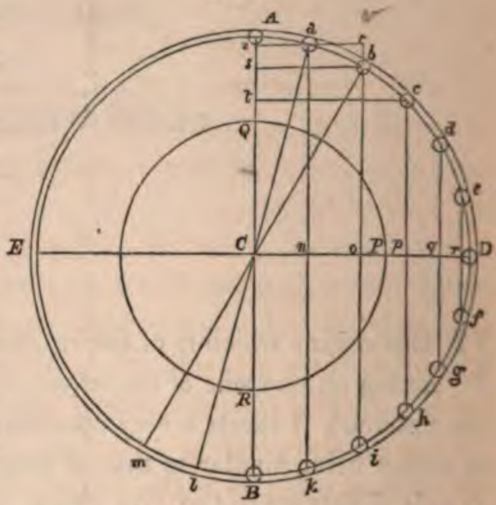
IN this engine which is of the rotatory kind, the motive power is a weight acting on one side of the wheel, where in consequence of the leverage thus obtained, it exerts a very considerable efficiency in producing rotation, the action being similar to that of water in the buckets of an overshot wheel; the only difference in the two cases, being, that in the water wheel, the weight and the leverage are constant for every revolution after the first, whereas in the case under consideration, the weight only is constant, the leverage being variable for every revolution.

The piston or weight which gives motion to the engine, is made to slide steam-tight in a cylinder placed in a central position across the wheel, and is continually raised from the lowest to the highest position by the admission of steam from the boiler, which flows through the hollow trunnions on which the wheel revolves, and making its way beneath the piston carries it to the upper side of the wheel, where, by its weight acting on a variable leverage it produces rotation, the steam which carried it up being allowed to make its escape in the atmosphere or a condenser, according to the nature of the construction.

In consequence of the variable leverage on which the weight acts, it will readily be admitted, that the working of this engine must be subject to considerable irregularity, and in consequence, a mean leverage must be adopted in calculating its power, the method of finding which, constitutes the only difficulty peculiar to the enquiry. It is however presumed, that the following exposition will render the matter perfectly intelligible, and prove that the calculation of the power of this kind of engine, is not more complex than that of the common beam engine where the steam acts at full pressure throughout the stroke. The method of finding the mean leverage is as follows.

When the cylinder is in the vertical position, and the piston or weight at the upper side of the wheel as represented in the marginal diagram, it can have no tendency to cause the wheel to revolve upon its axis; for since the action of the weight is wholly in the direction of gravity, the leverage in that position is nothing, and consequently the effect of the piston or weight to turn the wheel upon its axis is also nothing, because the direction of its action passes through the axis of rotation and can then have no influence whatever in communicating motion.

Fig. 15.



But no sooner does the cylinder assume an inclined position with the piston at its upper extremity, as represented by the oblique line *al*, than the weight takes effect, and by endeavouring to descend in the direction of gravity, it obtains a leverage, and operates to make the wheel revolve upon its axis, by acting at the extremity of the lever *az* or its equal C*n*.

When the cylinder is in the position indicated by the oblique line *bm*, the piston or weight is at *b*, and the leverage on which it acts, is *bs* or its equal C*o*, and when the weight is at *c* the leverage is *ct* or its equal C^{2}, the leverage being continually increased, until the cylinder arrives at the horizontal position ED, in which case the weight is at D, and its effect in producing rotation is then the greatest possible, the leverage being equal to the distance between the centre of the wheel and the centre of gravity of the piston or weight.

When the cylinder passes the horizontal position, where the piston acts with the greatest possible effect, the leverage on which it operates to produce rotation, continually decreases in passing through the second quadrant of revolution, in precisely the reverse order that it increased through the first, and when the cylinder comes into the vertical position, as represented by the vertical diameter AB, the weight is at the lowest possible point, and is then incapable of producing rotation, until it is again raised to the highest point by the admission of a fresh supply of steam from the boiler, when the same motion is repeated, the acting weight or piston being twice at the highest, and twice at the lowest point in one revolution of the wheel.

From the description here given, the general operation of the engine is manifest, and the mode of computing its power may be deduced from the following investigation, which determines the mean radius or leverage, and consequently the distance through which the weight acts at one revolution, and this is one of the chief elements employed in computing the power of this kind of engine.

Let ab in the preceding diagram, be an indefinitely small arc of the circumference, generated by the centre of gravity of the weight which gives motion to the wheel, and produce za to meet the vertical line ob in v. Then, because ab is conceived to be indefinitely small, the triangle abv may be considered as rectilinear; and since it is rightangled at v, it is similar to the triangle $b\mathrm{C}o$, and consequently, by the property of the triangle, the sides about the equal angles are proportional, and we have

$$bv : : ab, : \mathrm{C}o : : \mathrm{C}b;$$

and by equating the products of the extreme and mean terms, the value of bv becomes

$$bv = \frac{ab \times \mathrm{C}o}{\mathrm{C}b}.$$

Now, although the piston or weight has actually travelled over the arc ab, in the indefinitely small space of time allowed for the movement, yet the path or space described in the direction of the force, is only equal to vb or zs, which is the orthographic projection of the arc ab upon the vertical diameter of the wheel; and consequently it represents the space which the weight has descended in the direction of gravity. But by the principles of mechanics, the momentum or mechanical effect of the weight in moving through the indefinitely small arc ab

Is equal to the magnitude of the weight, multiplied by the space passed over in a direction perpendicular to the horizon.

In order therefore, to determine the mean effective radius of the wheel, we propose the following class of symbols, to represent the several quantities which enter the computation, it being necessary to carry out the investigation on the principles of analysis.

Let $Cb=r$, be the radius of the wheel, estimated from the centre of rotation to the centre of gravity of the weight,
$ab=\phi$, the indefinitely small arc passed over in any small space of time;
$vb=\delta$, the space passed over in the same time in the direction of gravity;
$w=$the actuating weight, or the weight of the piston estimated in pounds;
$m=$the momentum or mechanical effect in the small arc ab;
$l=Co$, the distance between the centre of the wheel and the ordinate bo;
and $\pi=$the circumference of a circle whose diameter is unity.

Then, by substituting the appropriate symbols in the foregoing value of vb, we obtain the following, viz.

$$\delta=\frac{l\phi}{r}.$$

This is the distance through which the weight acts in the direction of gravity, in passing over the small arc ab in the circumference; consequently, by the principle enunciated immediately above, if this distance be multiplied by the actuating power or weight, the momentum in that distance becomes

$$m=\frac{l\phi w}{r}.$$

In precisely the same manner it may be shown, that the momentum or mechanical effect produced by the weight in moving over any other arc, or falling through any other vertical distance or height,

> *Is equal to the weight or motive force, multiplied into the space through which it has descended in the direction of gravity.* Or, which is the same thing, *into the orthographic projection of the arc upon the vertical diameter of the wheel.*

So that when the force has acted through half a revolution, or when it has descended from the highest to the lowest point, the orthographic projection of the semicircle which it has traced in its descent, is equal to the vertical diameter of the wheel, estimated to the centre of gravity of the piston or weight which communicates the motion; hence we have by the above principle,

$$\text{momentum} = \text{weight} \times \text{diameter of the wheel.}$$

Let the semi-circumference of the circle, which is described by the centre of gravity of the weight, be divided into any number of small equal parts, in a, b, c, d, &c., each of the divisions or parts being indefinite in magnitude; and at the points of separation, let perpendiculars be drawn, to meet the horizontal radius CD in n, o, p, q, &c. Then it is obvious, that the distance from the centre of motion at C is continually varying as the arc increases, or according to the position of the point from which the perpendicular is drawn; so that in the expression

$$m = \frac{l\phi w}{r},$$

the symbol l is the only variable quality or element, by which the value of the momentum is affected.

Now, when the piston or weight is at A, or in its highest position, the value of l is nothing or zero, and the momentum of the weight is nothing also; for since there is no leverage, it can have no tendency to produce rotation, a circumstance which has already been alluded to; but when the weight arrives at D, that is, when the wheel revolves through one fourth part of its circuit, the value of l is the greatest possible, and is in that case equal to r, when it produces the greatest effect; and the expression for the maximum value of the momentum, is then

$$\text{momentum} = \text{weight} \times \text{arc of distance.}$$

But when the piston or weight passes the horizontal position the value of the leverage l again decreases, and continues to do so, until the weight has arrived at the lowest point, where the value of l is again equal to nothing, the wheel having then performed half a revolution.

It hence appears, that the momentum or mechanical effect of the moving piston or weight, passes through all possible magnitudes, from zero to ϕw, and again to zero twice in every revolution of the wheel; there must consequently be some intermediate value of l as CP, at which, if the weight acted uniformly, during half the revolution of the wheel, it would produce precisely the same mechanical effect as is produced by the variable action.

Let the symbol l in this state of things, be made to denote the mean radius of the wheel; then, if the weight w be supposed to act with uniform force at the distance CP, its momentum for the half revolution, will be

$$momentum = weight \times arc\ \mathrm{QPR}.$$

But the semi-circular arc QPR, is equal to the radius $\mathrm{CP} = l$ multiplied by the constant quantity 3.1416; that is, $\mathrm{QPR} = l\pi$, and if this be multiplied by the weight w, the momentum of the motive force in this case, becomes

$$momentum = 3.1416 \times mean\ radius \times weight.$$

But we have already seen that the momentum of the variable force is expressed by the weight multiplied by the diameter of the wheel reckoned to the centre of gravity of the motive power, and this momentum is the same in both cases; hence we get,

$$3.1416\, lw = 2\, rw,$$

consequently, by resolving this expression in reference to the symbol l, the value of the mean radius becomes

$$l = \frac{2\,r}{\pi} = \frac{2\,r}{3.1416} = 0.63662\, r.$$

Consequently, if the radius of the wheel, that is the distance between the centre of motion and the centre of gravity of the piston or weight, be multiplied by the constant decimal 0.63662, the product will be the mean radius, and if this be again multipled by 3.1416, the product will give the space traversed by the moving power for one semi-revolution of the wheel, or rather, the space through which it is supposed to act uniformly during one semi-revolution; and this is obviously equal to AB, the whole range of the centre of gravity of the motive power; but since this range is traversed twice during each complete revolution, the method of calculating the power of the engine becomes obvious. In order however to render the subject perfectly clear and intelligible to the general reader, it may be described in the following manner.

Find a point P in the radius of the wheel, at such a distance from the centre of motion, that if the weight were applied at that point, and acted uniformly throughout the revolution of the wheel, it would produce precisely the same ultimate effect as it actually produces with the variable leverage; and this point will be found to be distant from the centre of the wheel, by something less than two-thirds of the given radius or distance between the centre of motion and the centre of gravity of the motive power, the correct distance of the point being 0.63662 of the radius just specified. Therefore, if with the distance CP, equal to 0.63662 of CA, a circle be described, the circumference of that circle will be equal to the distance through which the uniform effective weight travels for each revolution of the wheel which carries the cylinder.

Now, taking the radius of the wheel from the centre of motion to the centre of gravity of the weight to be $4\frac{1}{2}$ feet, the mean effective radius CP will be 2.86479 feet, for $0.63662 \times 4\frac{1}{2} = 2.86479$. This therefore is the radius described by the uniform force, and consequently, the circumference or space passed over by the constant force will be $2.86479 \times 2 \times 3.1416 = 18$ feet; therefore, by taking the number of revolutions at 30 per minute, we have $18 \times 30 = 540$ feet per minute for the velocity of the moving power or weight.

Now, supposing this weight or power to be somewhere about 400 pounds, and the power of the engine will be $540 \times 400 = 216000$ pounds; and dividing this by 33000 pounds, which constitutes the measure of a horse's power, the nominal power of the engine in horses' power becomes

$$216000 \div 33000 = 6.575 \text{ horses nearly.}$$

On the above principle may the power of any engine of this kind be computed; but to establish the correctness of the principle, we shall give the calculation of an engine constructed on this system, and which is actually at work, driving a fan blast, at Redruth in Cornwall; the coincidence of the calculated power with that deduced from experiment, may be the means of establishing the correctness of both, and enforcing conviction as to the validity of the theory here investigated.

In the case here alluded to, the total diameter of the wheel is 9 feet, the range of the piston 7 feet, the diameter of the cylinder 7 inches, and the wheel making 24 revolutions per minute.

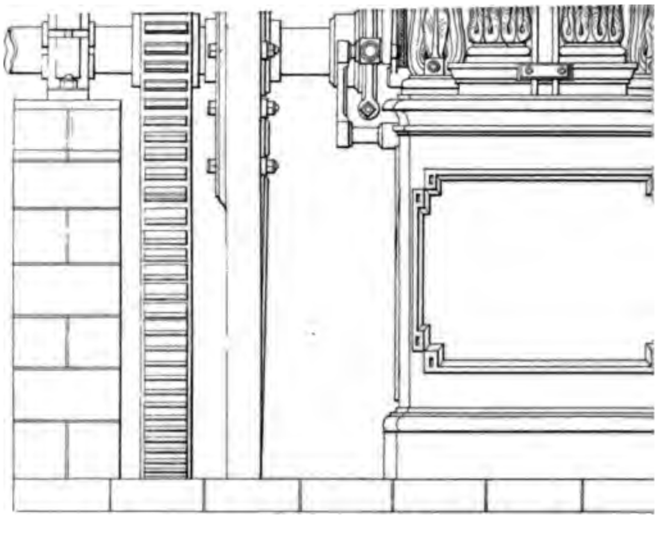
Since the range of the piston is 7 feet, let this be multiplied by the constant 0.63662, and we get 4.45634 for the mean diameter of revolution, and consequently, the circumference or distance traversed by the constant force at every revolution of the wheel, is $4.45634 \times 3.1416 = 14$ feet; but the wheel makes 24 revolutions per minute; consequently, the distance through which the constant force moves in one minute, becomes $14 \times 24 = 336$ feet; but the weight is equal to 500 pounds; therefore, the momentum or power of the engine, is

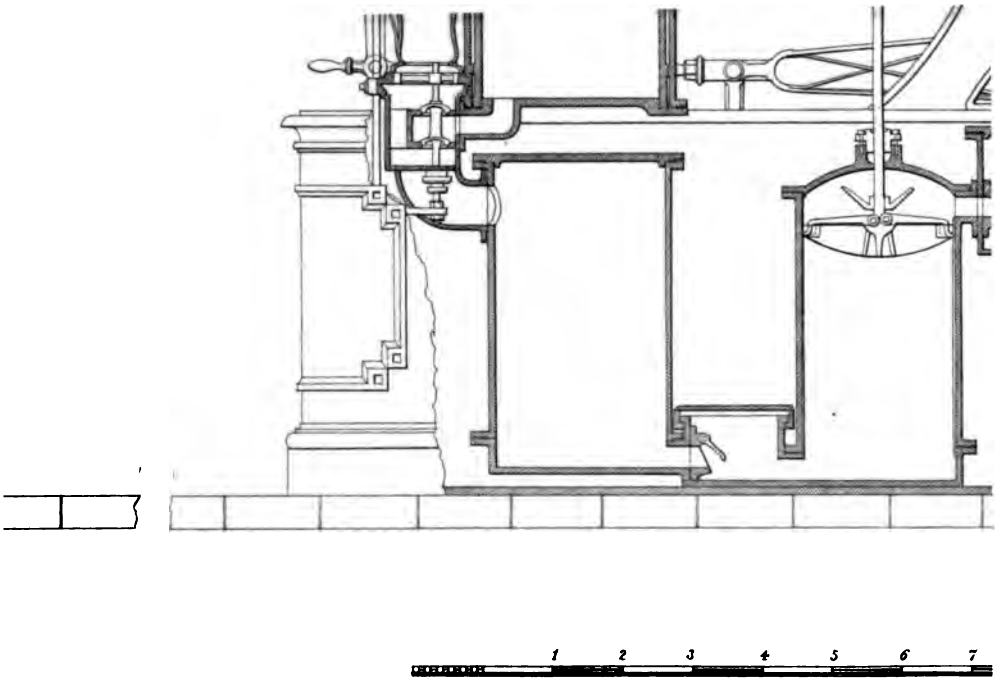
$$336 \times 500 \div 33000 = 5 \text{ horses.}$$

This kind of engine, with some trifling alterations, such as supplying a curved guide to prevent the piston from striking violently against the inner part of the wheel, has by experience, been found to work with greater ease and economy than any other kind of engine now extant, and when some other improvements that the inventor contemplates have been added, it will work with still greater advantages.

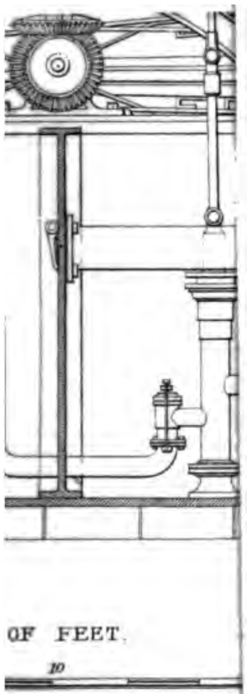
It will be observed that in this kind of engine, the force of the steam is only employed to lift the piston or weight from the bottom to the top of the wheel, which by disregarding the effects of friction, will not require more than 12 or 13 lbs. pressure upon the square inch.

London:—Printed by R. B. Shaw, Old Kent Road.

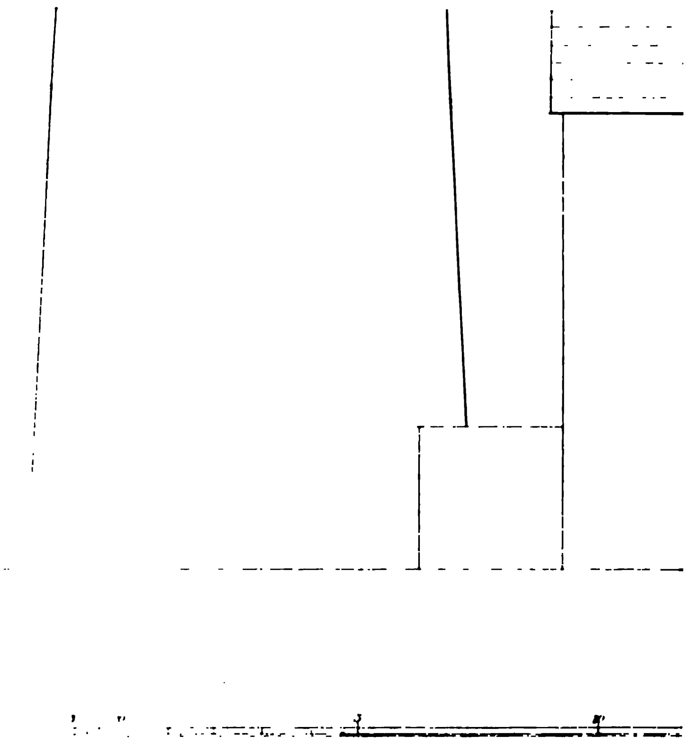


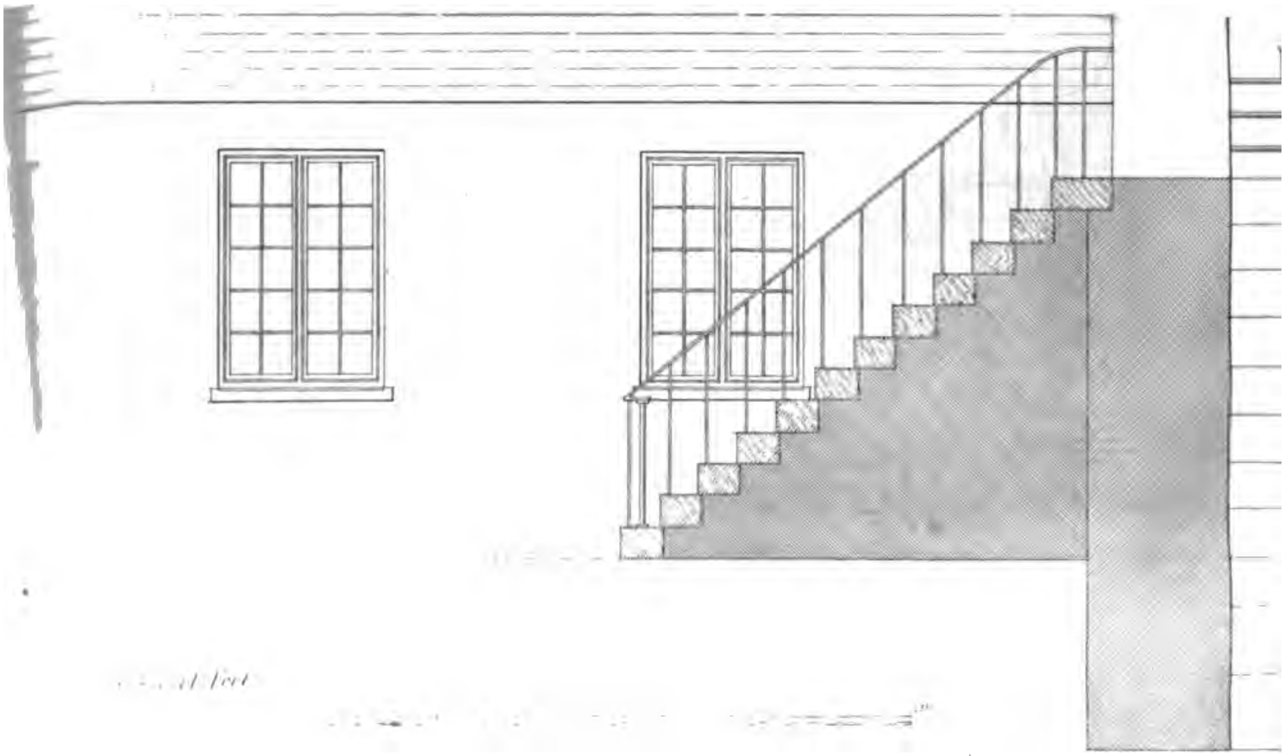


J:

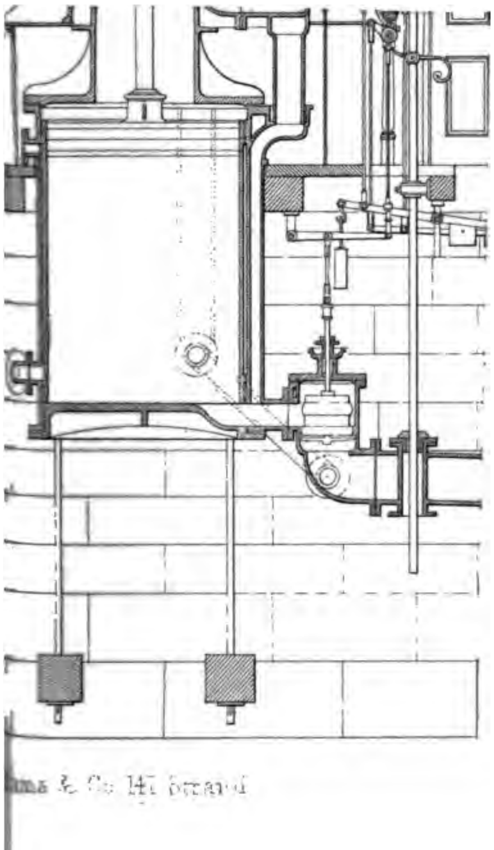


s & Co., 141 Strand.





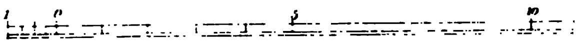
Feet
10

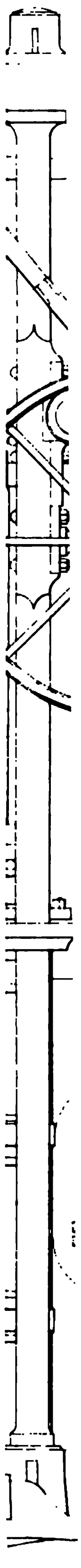


ms & Co 141 Strand

SIMS'S PATENT CON

DELABO





OF

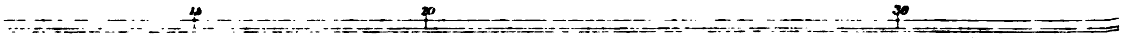
IBINED WINDING STEAM EN

ERECTED AT THE

LE SLATE WORKS,

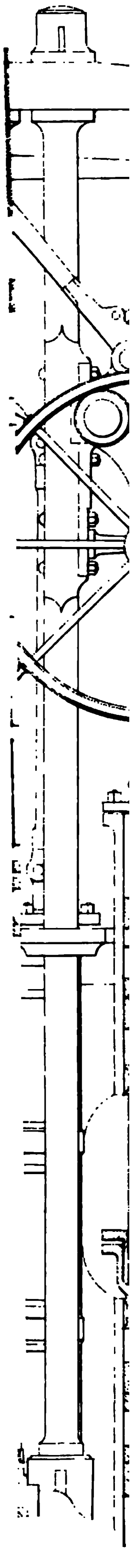
BY WILLIAM WEST,
Memr. Inst. Civil Engineers.

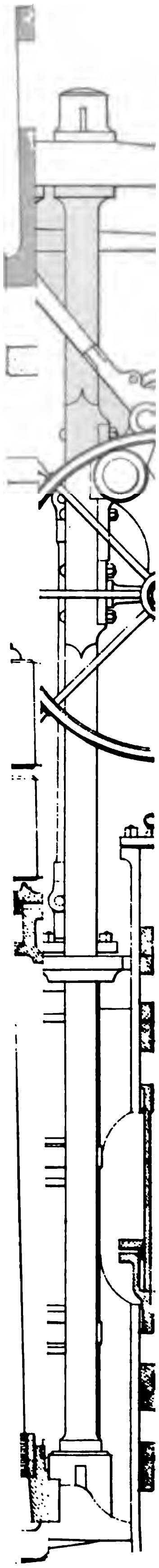
Scale of Feet.

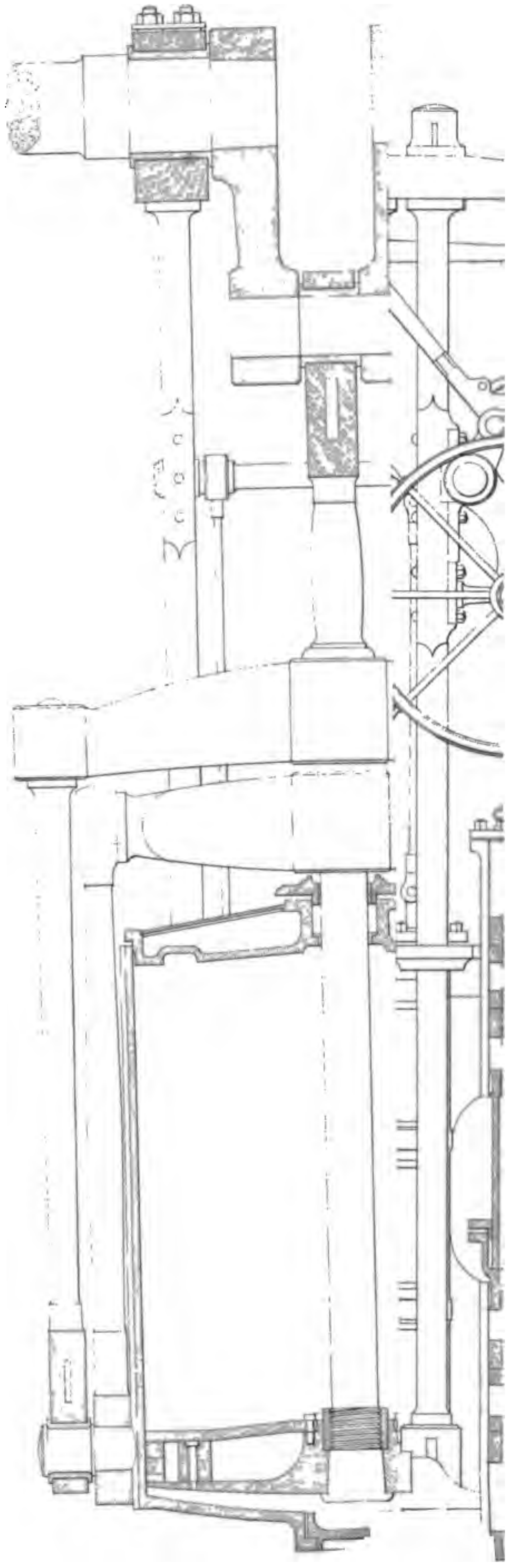


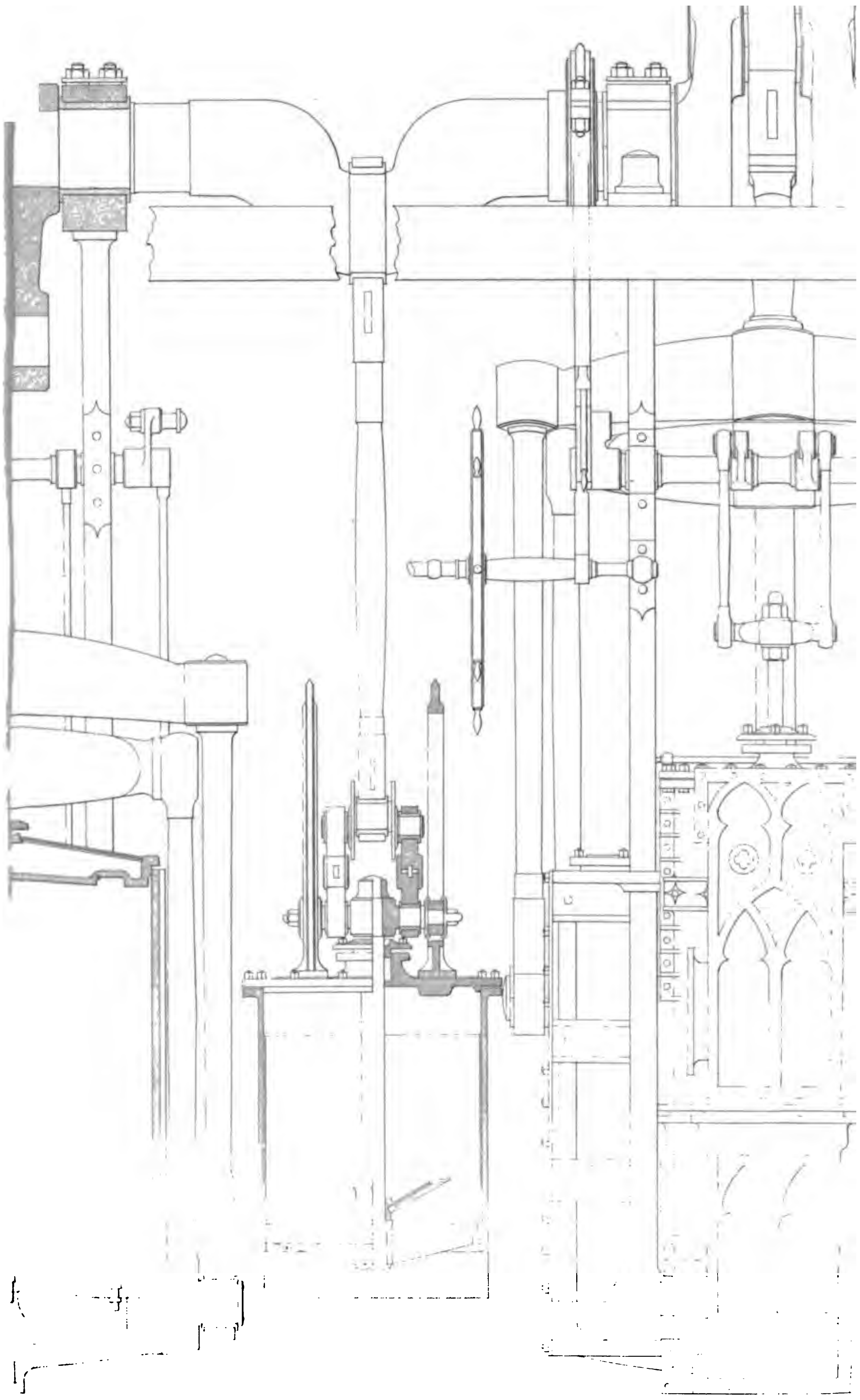
John Williams & Co. 141, Strand.

NE,



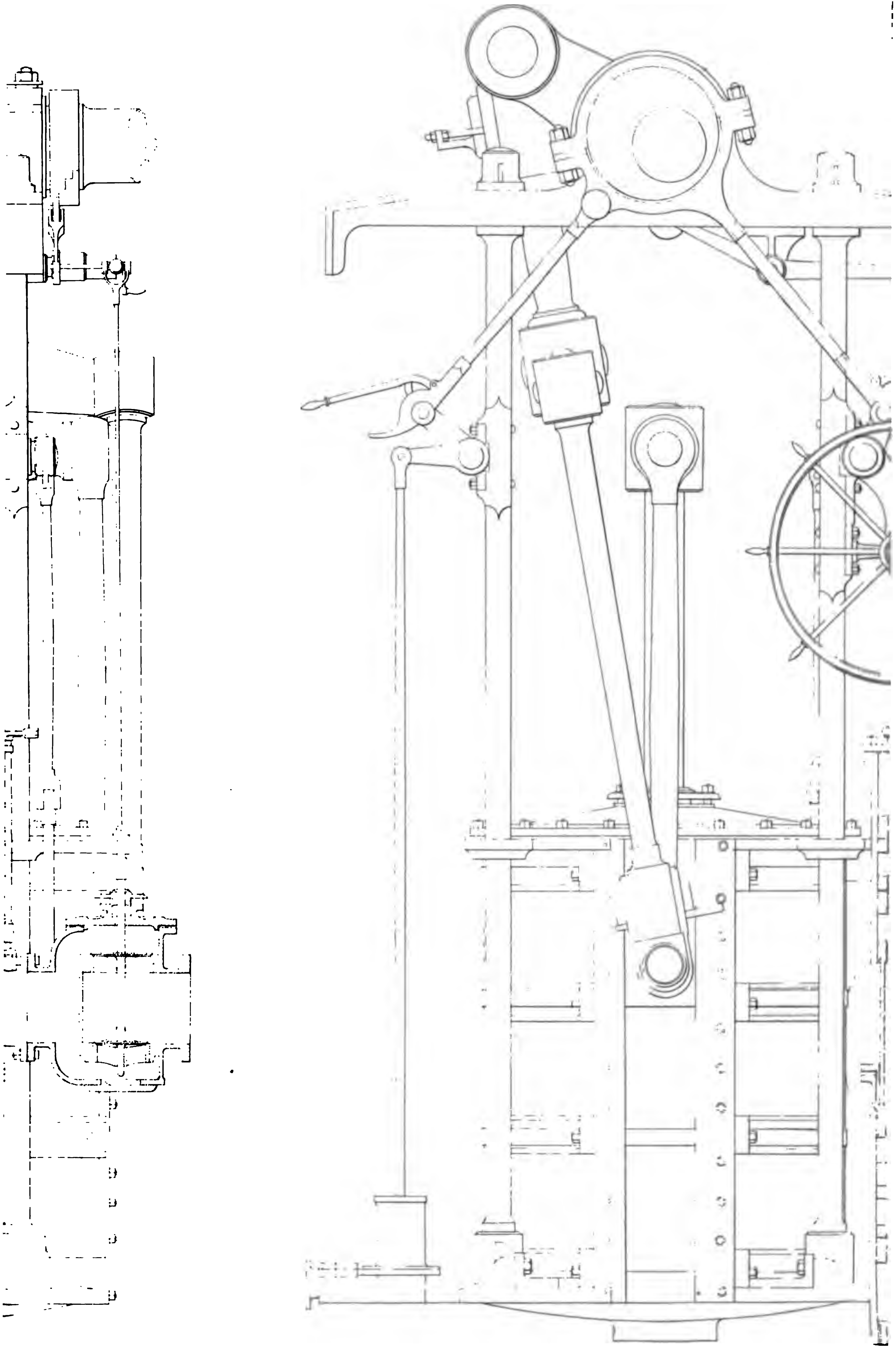


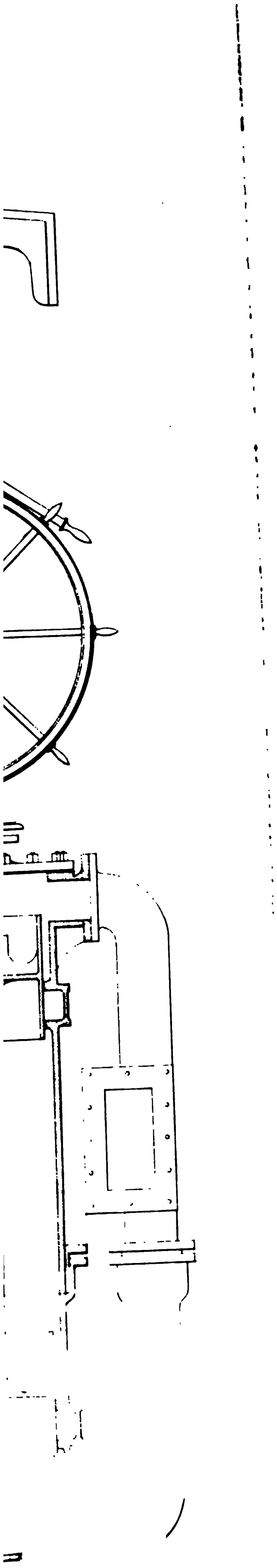


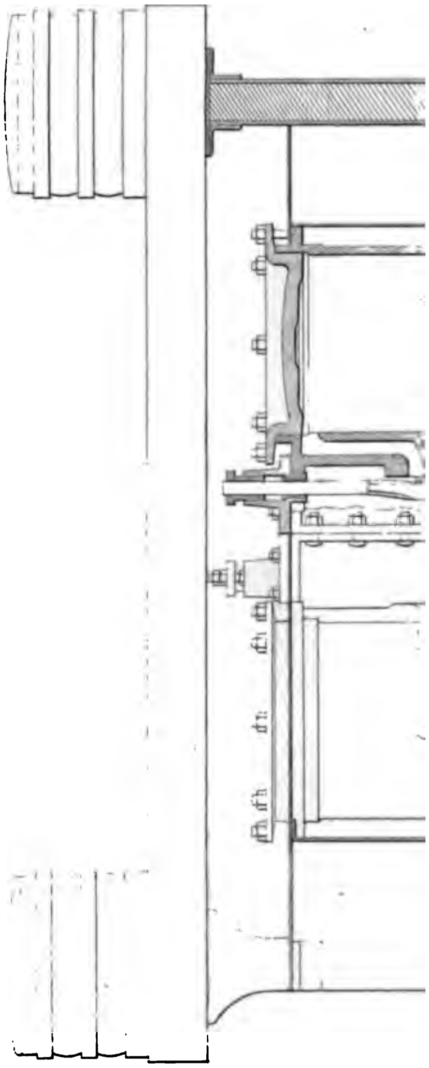


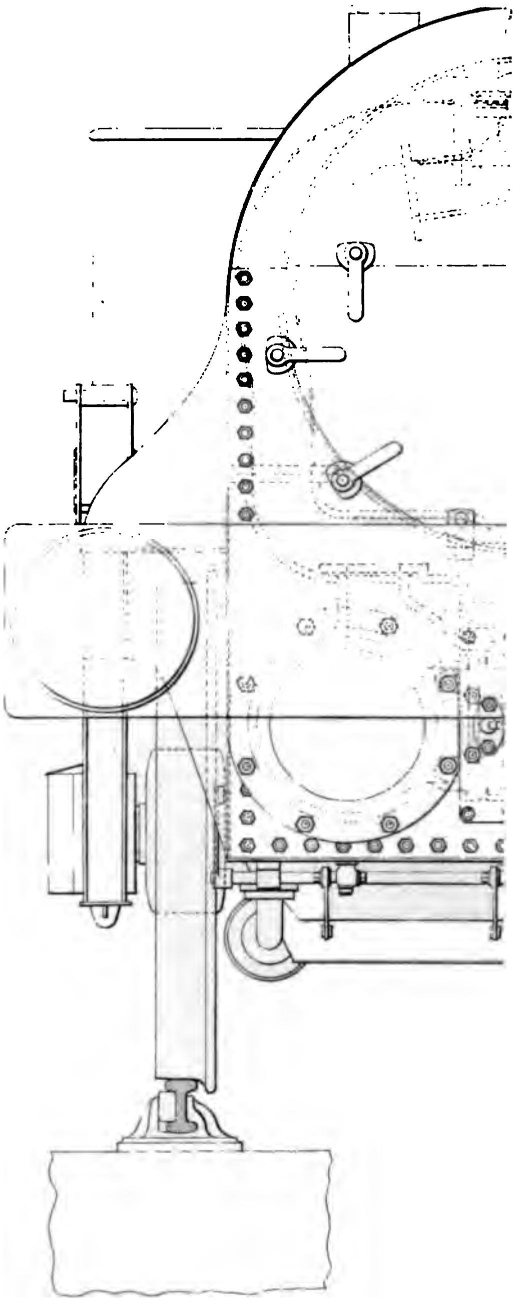
London Published by John W

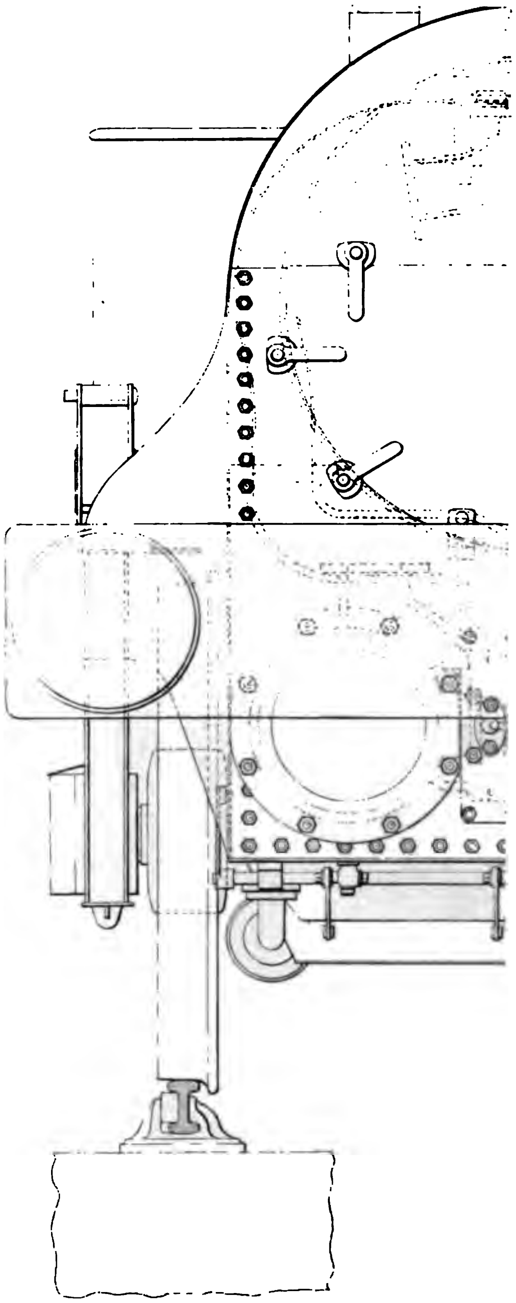
& Mc Gregor, Glasgow.











· STANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STAN
D UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD UNIV
Y LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBR
IVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY
IBRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES
TANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD
· STANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STAN
D UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD UNIV
Y LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBR
IVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY
IBRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES
TANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD
· STANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STAN
RD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD UNIV
ITY LIBRARI TANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBR

TJ 464 .H688
Analytical principles and prac C.1
Stanford University Libraries
3 6105 035 198 303

TJ
46
H68

Stanford University Libraries
Stanford, California

Return this book on or before date due.